Weighted Hermite-Hadamard's inequality without symmetry condition for fractional integral

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Abstract

Weighted Hermite-Hadamard's inequality without symmetry condition for fractional integral is discussed. The main results of this paper improve and generalize some previous results obtained by many researchers.

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1 Preliminaries and some results

One of the most well-known inequalities for the class of convex functions is the Hermite-Hadamard inequality given in [4]

$$\frac{f\left(a+b\right)}{2} \leqslant \frac{1}{b-a}\int_a^b f\left(x\right)dx \leqslant \frac{f\left(a\right)+f\left(b\right)}{2}, \tag{1}$$

which plays an important role in nonlinear analysis. Weighted generalization of 1 based on the symmetry condition was proved by Fejér [2].

Theorem 1.1. *[2] If $f : [a,b] \to \mathbb{R}$ is a convex function, then the following inequality holds*

$$\frac{f\left(a+b\right)}{2}\int_a^b \omega\left(x\right)dx \leqslant \int_a^b \omega\left(x\right)f\left(x\right)dx \leqslant \frac{f\left(a\right)+f\left(b\right)}{2}\int_a^b \omega\left(x\right)dx$$

where $\omega : [a,b] \to (0,\infty)$ is a non-negative function which is integrable and symmetric about $\frac{a+b}{2}$.

However, the lack of symmetry condition in many problems in statistics, probability and engineering is reasonable. Therefore, finding a weighted generalization of Hermite-Hadamard's inequality without the symmetry condition is interesting for researchers [1].

Theorem 1.2. *[1] Let $f : [a,\, b] \subset (0,\infty) \longrightarrow \mathbb{R}$ be a differentiable function and $\omega : [a,\, b] \longrightarrow (0,\, \infty)$ be an integrable function.*
(i) If the function $\frac{f'}{\omega}$ is increasing, then the following inequality is hold,

$$\frac{\int_a^b \omega\left(x\right)f\left(x\right)dx}{\int_a^b \omega\left(x\right)dx} \leqslant \frac{f\left(a\right)+f\left(b\right)}{2} \tag{2}$$

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