New rational approximation of Mittag-Leffler function with orthogonal polynomials

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Abstract

In this paper we drive a uniform rational approximation for the Mittag-Leffler function using the Chebyshev polynomials and asymptotic series. Next, we use this approximation to find the solution of the fractional diffusion equation.

Keywords: Mittag-Leffler function, global rational approximation, Time- Fractional Diffusion Equation

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1 Introduction and Preliminaries

The Mittag-Leffler functions arise naturally as the solution of fractional differential and integral equations. The Mittag-Leffler function of order α is stated as the following series [3]

$$E_\alpha(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(1+\alpha k)}, \tag{1}$$

where α is an arbitrary real number. For computational works, one have to truncate the above series which yields truncated error cost in computation. So it is important to substitute a good approximation instead of the Mittag-Leffler function expansion (1). The Pade approximations for the Mittag-Leffler function are discussed in [4]. Atkinson et. al. used both Taylor and asymptotic series to find good approximations for the Mittag-Leffler function [1]. In this paper we introduce a new method based on [1] to approximate the Mittag-Leffler function. In this method we substitute the Chebyshev polynomial expansion instead of (1) to obtain a better approximation for $E_\alpha(-x)$ in two cases.

Case 1. Expanding $E_\alpha(-x)$ by the Chebyshev polynomials of the first kind. In this case we have

$$\Gamma(1-\alpha)xE_\alpha(-x) = \Gamma(1-\alpha)x\sum_{k=0}^{m-2} a_k T_k(x-1) + O(x^m) \equiv a(x) + O(x^m), x \in [0,a]. \tag{2}$$

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