46th Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University

A greedy meshless method for solving boundary value problems

Yasin Fadaei*
Shahid Bahonar University of Kerman

Mahmoud Mohseni Moghadam
Shahid Bahonar University of Kerman

Abstract

In this paper we use a meshless method based on a greedy algorithm to solve boundary value problems (BVPs). This method is greedy Kansa's method that use the optimal trial points. In the greedy algorithm , the optimal trial points for interpolation obtained among a huge set of initial points are used for numerical solution of BVPs. This paper shows that selection nodes greedily yields the better conditioning and good approximation in contrast with the usual Kansa method. A well known BVP is solved and compared with the usual Kansa's method.

Keywords: Greedy algorithm, Meshless method, Radial basis function
Mathematics Subject Classification [2010]: 65N35, 65N22

1 Introduction

A greedy algorithm is an algorithm that follows the problem solving heuristic of making the locally optimal choice at each stage with the hope of finding a global optimum. A recent survey of the approximation properties of such algorithms is given in [1]. Schaback and Muller [3] has shown that representations of kernel-based approximants in terms of the standard basis of translated kernels are notoriously unstable. They introduced the Newton bases functions with a recursively computable set of basis functions and vanishing at increasingly many data points turn out to be more stable. In [4] adaptive calculation of Newton bases is used, which turns out to be stable, complete, orthonormal computable. In this work, we will apply the greedy method to meshless method for solving a linear PDE problem is given in the form

$$\begin{aligned} Lu &= f, \quad in\ \Omega, \\ Bu &= g \quad on\ \partial\Omega \end{aligned} \tag{1}$$

with a linear differential operator L and a linear boundary operator B. Consider smooth symmetric positive definite kernel $K : \Omega \times \Omega \to \mathbb{R}$ on spatial domain Ω. This means that for all finite sets $X := \{x_1, \cdots x_N\} \subseteq \Omega$ the kernel matrix $A := (K(x_j, x_k))_{1\leq j,k\leq N}$ is symmetric and positive definite. It is well-known that this kernel is reproducing in a "native" Hilbert space $\mathcal{N}_k = \overline{\text{span}\{K(x,.) : |x \in \Omega\}}$ of functions on Ω in the sense $\langle u, K(x,.)\rangle_{\mathcal{N}_k} = u(x) \ \ \forall x \in \Omega, \ \forall u \in \mathcal{N}_k$.

*Speaker