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Primary Decomposition of Ideals in MV-algebras

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Abstract

In this paper, we investigate the ideal theory in MV-algebras and we define the notions of implicative MV-algebras and primary (P-primary) ideals in MV-algebras. Then we show that in implicative MV-algebras, if an ideal has a primary decomposition, then it has a reduced primary decomposition.

Keywords: MV-algebra, radical, primary and P-primary ideals, primary decomposition
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1 Introduction

MV-algebras were defined by C.C. Chang [1, 2] as algebras corresponding to the Lukasiewicz infinite valued propositional calculus. These algebras have appeared in the literature under different names and polynomially equivalent presentation: CN-algebras, Wajsberg algebras, bounded commutative BCK-algebras and bricks. The notion of prime ideal in an MV-algebra was introduced by Chang. Since the notion of ideal in MV-algebras is important, for completion of study of ideals in MV-algebras, in this paper, we present definitions of radical of an ideal and primary decomposition of an ideal.

Definition 1.1. [3] An *MV-algebra* is a structure $M = (M, \oplus, ', 0)$ of type $(2, 1, 0)$ such that:
$(MV1)$ $(M, \oplus, 0)$ is an Abelian monoid,
$(MV2)$ $(a')' = a$,
$(MV3)$ $0' \oplus a = 0'$,
$(MV4)$ $(a' \oplus b)' \oplus b = (b' \oplus a)' \oplus a$,
If we define the constant $1 = 0'$, then operations $\odot$ and $\ominus$ are defined by $a \odot b = (a' \oplus b')'$, $a \ominus b = a \odot b'$. Also, operations $\vee$ and $\wedge$ on M are defined by $a \vee b = (a \odot b') \oplus b$ and $a \wedge b = a \odot (a' \oplus b)$, for every $a, b \in M$. An ideal of MV-algebra M is a subset I of M, satisfying the following condition: $(I1)$ $0 \in I$, $(I2)$ $x \leq y$ and $y \in I$ implies that $x \in I$, $(I3)$ $x \oplus y \in I$, for every $x, y \in I$. We let $\mathcal{I}(M)$ be the set of all ideals of M. A proper ideal P of M is a prime ideal if for $x, y \in M$, $x \wedge y \in P$ implies $x \in P$ or $y \in P$. Equivalently, P is prime if and only if $x \ominus y \in P$ or $y \ominus x \in P$, for every $x, y \in M$.

Note: From now on, in this paper, we let M be an MV-algebra and $\mathcal{PI}(M)$ be the set of all prime ideals of M.

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