A compact finite difference method without using Hopf-Cole transformation for solving 1D Burgers' equation

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Abstract

A new compact finite difference (CFD) method for solving one-dimensional (1D) Burgers' equation without using the Hopf-Cole transformation is analyzed. This method leads to a system of linear equations involving tridiagonal matrices and the rate of convergence of the method is of order $O(k^2+h^4)$ where k and h are the time and space step sizes, respectively. Numerical results obtained by the proposed method are compared with the exact solutions and the results obtained by some other methods.

Keywords: Burgers' equation, compact finite difference method
Mathematics Subject Classification [2010]: 65M06, 65M12

1 Introduction

Burgers' equation was formulated by Bateman in 1915 [2] and later treated by Burgers [3]. This equation is also called the nonlinear advection-diffusion equation, and can be regarded as a qualitative approximation of the Navier-Stocks equations. Recently, Xie et al. [4] applied the Hopf-Cole transformation method to linearize the equation and constructed a CFD method which is unconditionally stable and its accuracy is second- and fourth-order accurate in time and space, respectively. We aim to construct a CFD method for the 1D Burgers' equation without using the Hopf-Cole transformation.

2 Construction of the method

We consider the following one-dimensional nonlinear Burgers' equation

$$u_t + uu_x - \nu u_{xx} = 0, \qquad a < x < b, \quad 0 < t < T, \tag{1}$$

where $\nu = 1/Re$ in which Re is the Reynolds' number. The following boundary and initial conditions are also considered

$$u(a,t) = 0, \quad u(b,t) = 0, \qquad 0 \le t \le T, \qquad u(x,0) = f(x), \qquad a \le x \le b,$$

where f is a given function. In order to construct a CFD method, we select integers $M, N > 0$ and define $h = (b-a)/M,\ k = T/N$. The grid points for this situation are

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