C*-Algebras and Dynamical Systems, a Categorical Approach

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Abstract

There are interactions between C*-algebras, essentially minimal dynamical systems, ordered Bratteli diagrams, and dimension groups. We extend these interactions to encompass morphisms of these categories. We show that the category of essentially minimal dynamical systems is equivalent to the category of essentially simple ordered Bratteli diagrams. Especially, one can describe the factors of certain dynamical systems using a graphical approach. The functor K^0 is constructed to distinguish various types of orbit equivalence. Relations with crossed products of C*-algebras are studied.

Keywords: C*-algebra, ordered Bratteli diagram, essentially minimal system, category, dimension group
Mathematics Subject Classification [2010]: 46L05, 37B05, 37A20.

1 Introduction

In 1972, Bratteli in a seminal paper introduced what are now called Bratteli diagrams to study AF algebras [3]. He associated to each AF algebra an infinite directed graph, its Bratteli diagram, and used this very effectively to study AF algebras. In 1976, based on the notion of a Bratteli diagram, Elliott introduced dimension groups and gave a classification of AF algebras using K-theory [4]. In fact, he showed that the functor $\mathrm{K}_0 : \mathbf{AF} \to \mathbf{DG}$, from the category of AF algebras to the category of dimension groups is a strong classification functor [4, 5].

In [1], the authors introduced an appropriate notion of morphism between Bratteli diagrams and obtained the category of Bratteli diagrams, $\mathbf{BD}$, such that isomorphism of Bratteli diagrams in this category coincides with Bratteli's notion of equivalence. We showed that the map $\mathcal{B} : \mathbf{AF} \to \mathbf{BD}$, defined by Bratteli on objects, is in fact a functor. The fact that this is a strong classification functor [1, Theorem 3.11], is a functorial formulation of Bratteli's classification of AF algebras and completes his work from the classification point of view introduced by Elliott in [5].

In a different direction, Bratteli diagrams were used to study certain dynamical systems. In 1981, A. V. Versik used Bratteli diagrams to construct so-called adic transformations [8]. Based on his work, Herman, Putnam, and Skau introduced the notion of an ordered Bratteli diagram and associated a dynamical system to each (essentially simple) ordered Bratteli diagram [7]. They showed that there is a one-to-one correspondence between essentially simple ordered Bratteli diagrams and essentially minimal dynamical

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