Torsion theory cogenerated by a class of modules

Behnam Talaee*
Babol University of Technology

Abstract

We introduce and study a generalization of a class of modules related to radical. The torsion theory cogenerated by this class of modules will be investigated in this paper. We will show that the module $N \in \sigma[M]$ is $M-$radical if and only if For any $M-$ injective module I and any homomorphism $f : N \longrightarrow I$ in $\sigma[M]$, we have $Im(f) \subseteq Rad(I)$. Also we conclude that $N = Re_{Rd[M]}(N)$ if and only if for every nonzero homomorphism $f : N \longrightarrow K$ in $\sigma[M]$, $Im(f) \nsubseteq Rad(K)$, where $Rd[M]$ is the class of all $M-$radicla modules. The relationship between this modules and some other kind of modules will be studied.

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1 Introduction

Throughout this article, all rings are associative and have an identity, and all modules are unitary right modules.

$N \subseteq^{\oplus} M$ means that N is a direct summand of M. A submodule L of M is called *small* in M (denoted by $L \ll M$) if, for every proper submodule K of M, $L+K \neq M$. The sum of all small submodules of M is called the *radical* of M and is denoted by $Rad(M)$.

A submodule N of M is called *essential* in M (denoted by $N \subseteq^{ess} M$) if $N \cap K \neq 0$ for every nonzero submodule K of M.

Let M be a module and $B \leq A \leq M$. If $A/B \ll M/B$, then B is called a *cosmall* submodule of A in M. The submodule A of M is called *coclosed* if A has no proper *cosmall* submodule. Also B is called a *coclosure* of A in M if B is a cosmall submodule of A and B is coclosed in M.

For a module M, an injective module E is called an *injective envelope (or injective hull)* of M if, $M \subseteq^{ess} E$. It is well known that for every ring R, every $R-$module has injective envelope. We refer for more information and basic notations to [1].

Let $\mathbb{A}$ be a nonempty class of modules in $\sigma[M]$. Recall the following classes

$$\mathbb{A}^{\circ} = \{B \in \sigma[M] | Hom(B, A) = 0; \forall A \in \mathbb{A}\} = \{B \in \sigma[M] | Re(B, \mathbb{A} = B\}$$

*Speaker