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An implicit finite difference method for solving integro-partial time fractional diffusion equation with weakly singular kernel

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Abstract

In this paper we develop an implicit finite difference method to solve an one-dimensional linear integro-partial time fractional diffusion equation with weakly singular kernel, formulated with Caputos fractional derivative. The numerical test is performed and comparative results are provided to illustrate the usefulness of the proposed method.

Keywords: Implicit finite difference method, Fractional calculus, Numerical methods, Time fractional diffusion equation
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1 Introduction

In this paper we introduced an method for the numerical solution of the following linear integro-partial time fractional diffusion equation with weakly singular kernel:

$$\frac{\partial^\alpha u}{\partial t^\alpha}(x,t) = \mu \frac{\partial^2 u}{\partial x^2}(x,t) + \int_0^t (t-s)^{-1/2} \frac{\partial^2 u}{\partial x^2}(x,s)\mathrm{d}s, \qquad x \in [0,1], t \in [0,T], \tag{1}$$

where $\mu \geqslant 0, 0 < \alpha \leqslant 1$ and the unknown real function $u(x,t)$ is sought for $0 \leqslant x \leqslant 1, 0 \leqslant t \leqslant T$, with the boundary and initial conditions:

$$u(0,t) = u(1,t) = 0, \qquad t \geqslant 0, \qquad u(x,0) = g(x), \qquad 0 \leqslant x \leqslant 1. \tag{2}$$

Definition 1.1. The Riemann-Liouville fractional integral operator of order $\alpha \geqslant 0$ of a function $f(t)$ with respect to point $t = 0$ is defined as:

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau)\mathrm{d}\tau, \quad \alpha > 0, t > 0, \qquad I^0 f(t) = f(t).$$

Definition 1.2. The Caputo fractional derivative of order α of function$f(t)$ is defined as:

$$D_*^\alpha f(t) = I^{n-\alpha} D^n f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\mathrm{d}\tau, \quad n \in \mathbb{N}, \qquad n-1 < \alpha \leqslant n.$$

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