46th Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University

Twin 2-rainbow dominating sets in graphs

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Abstract

A 2-rainbow dominating function ($2RDF$) of a graph G is a function f from the vertex set $V(G)$ to the set of all subsets of the set $\{1,2\}$ such that for any vertex $v \in V(G)$ with $f(v) = \emptyset$ the condition $\sum_{u \in N(v)} f(u) = \{1,2\}$ is fulfilled, where $N(v)$ is the open neighborhood of v. The weight of a $2RDF$ is the value $w(f) = \sum_{v \in V(G)} |f(v)|$. The 2-rainbow domination number of a graph G, denoted by $\gamma_{r2}(G)$, is the minimum weight of a $2RDF$ of G. In this paper, for a directed graph D we define twin 2-rainbow dominating function in which a vertex of label $\emptyset$ has $\{1,2\}$ both in its in-neighbourhood and its out-neighbourhood. We investigate it for some well-known graphs and then obtain a Nordhaus Gaddum inequality for the twin 2-rainbow domination number. Also, we provide upper bounds on this parameter in terms of the diameter of the graph.

Keywords: 2-rainbow domination, cartesian product, Harary graphs, Petersen graphs, Nordhaus Gaddum inequality
Mathematics Subject Classification [2010]: 13D45, 39B42

1 Introduction

For the basic terminology on graphs and digraphs (directed graphs) we refer the reader to [2]. Rainbow domination and other related concepts have been widely studied for undirected graphs, see [1] and [6]. The respective analogues on directed graphs however have not received the same amount of interest.
A function $f : V(G)(\{1, ..., k\})$ is called a k-rainbow dominating function (for short $kRDF$) of G if $\sum_{u \in N(v)} f(u) = \{1, ..., k\}$ for each vertex $v \in V(G)$ with $f(v) = \emptyset$. By $w(f)$ we mean $\sum_{v \in N(v)} |f(v)|$ and we call it the weight of a k-rainbow dominating function f in G. The minimum weight of a $kRDF$ of G is called the k-rainbow domination number of G and it is designated by $\gamma_{rk}(G)$. An assignment f is called a γ_{rk}-function if it is a $kRDF$ of G and $w(f) = \gamma_{rk}(G)$. For more information about k-rainbow dominating functions consult [3] and [5].
We consider the case $k = 2$ in this paper. The 2-rainbow dominating functions are extensively studied in recent literature. Here we define twin 2-rainbow dominating function and study the parameter for complete graphs, paths, cycles, Harary graphs and Petersen graphs. A similar definition, so-called twin dominating function, has been already offered for graphs. Refer to [4].

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