Two-stage waveform relaxation method for linear system of IVPs with non-constant HPD coefficients

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Abstract

In this paper, a two-stage waveform relaxation method is introduced to solve the system of initial value problems in the form $y'(t) + A(t)y(t) = f(t)$. Convergence of this method is analyzed when $A(t)$ is Hermitian positive definite matrix for every $t \in [t_0, T]$. Finally, a numerical example is presented to illustrate efficiency of the method.

Keywords: Two-stage method, Waveform relaxation, Hermitian positive definite, P-regular splitting.
Mathematics Subject Classification [2010]: 13D45, 39B42

1 Introduction

In [3], the two-stage waveform relaxation (TSWR) method was applied to solve the linear system of ordinary differential equations $y'(t) + Ay(t) = f(t)$, where A is an M-matrix. Indeed this method was obtained by combining the waveform relaxation (WR) method with two-step iterative strategy. Afterwards, in [2, 4] the TSWR was investigated to solve linear systems of ordinary differential equations (ODEs) and differential-algebraic equations, when the coefficient matrices are Hermitian positive definite and Hermitian positive semi-definite. Recently the TSWR method has been applied to solve the linear system of ODEs

$$\begin{cases} y'(t) + A(t)y(t) = f(t), \\ y(t_0) = y_0, \quad t \in [t_0, T], \end{cases} \tag{1}$$

where $A(t) : [t_0, T] \longrightarrow \mathbb{C}^{m\times m}$ is a nonsingular M-matrix for every $t \in [t_0, T]$ with continuous entries and $f(t) : [t_0, T] \longrightarrow \mathbb{C}^m$ is supposed to be continuous (see [1]). In this paper, we study the WR and TSWR methods for (1), when $A(t)$ is Hermitian positive definite for every $t \in [t_0, T]$. We will use the notation $A(t) \succ 0$ ($A(t) \succeq 0$) for a matrix function $A(t)$ to be Hermitian positive (semi-)definite for every $t \in [t_0, T]$.

Definition 1.1. The splitting $A(t) = C(t) - D(t)$ is called P-regular if $C^H(t) + D(t) \succ 0$, and Hermitian P-regular splitting if $C(t) \succ 0$ and $D(t) \succeq 0$.

Definition 1.2. We say that the splitting $A(t) = M(t) - N(t) - D(t)$ is composite P-regular if $C(t) = M(t) - N(t)$ and $A(t) = C(t) - D(t)$ are both P-regular splittings, and a composite Hermitian P-regular splitting if $M(t) \succ 0$, $N(t) \succeq 0$ and $D(t) \succeq 0$.

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