On a conjecture of Richard Stanley*

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Abstract

Let $\mathbb{K}$ be a field and $S = \mathbb{K}[x_1, \dots, x_n]$ be the polynomial ring in n variables over the field $\mathbb{K}$. In 1982, Stanley defined what is now called the Stanley depth of a multigraded S-module. He conjectured that Stanley depth is an upper for the depth of the module. This conjecture has been recently disproved by Duval et al., [2]. In this talk, we describe their counterexample. We also present the recent developments in this topic.

Keywords: Stanley depth, Monomial ideal, Cohen-Macaulay simplicial complex, Partitionable simplicial complex
Mathematics Subject Classification [2010]: 13C15, 13C13, 05E40

1 Introduction

Let $\mathbb{K}$ be a field and let $S = \mathbb{K}[x_1, \dots, x_n]$ be the polynomial ring in n variables over $\mathbb{K}$. Let M be a finitely generated $\mathbb{Z}^n$-graded S-module. Let $u \in M$ be a homogeneous element and $Z \subseteq \{x_1, \dots, x_n\}$. The $\mathbb{K}$-subspace $u\mathbb{K}[Z]$ generated by all elements uv with $v \in \mathbb{K}[Z]$ is called a *Stanley space* of dimension $|Z|$, if it is a free $\mathbb{K}[Z]$-module. Here, as usual, $|Z|$ denotes the number of elements of Z. A decomposition $\mathcal{D}$ of M as a finite direct sum of Stanley spaces is called a *Stanley decomposition* of M. The minimum dimension of a Stanley space in $\mathcal{D}$ is called the *Stanley depth* of $\mathcal{D}$ and is denoted by sdepth($\mathcal{D}$). The quantity

$$\mathrm{sdepth}(M) := \max\big\{\mathrm{sdepth}(\mathcal{D}) \mid \mathcal{D} \text{ is a Stanley decomposition of } M\big\}$$

is called the *Stanley depth* of M. For a reader friendly introduction to Stanley depth, we refer to [7] and for a nice survey on this topic, we refer to [3].

A $\mathbb{Z}^n$-graded S-module M is said to satisfies *Stanley's inequality* if

$$\mathrm{depth}(M) \leq \mathrm{sdepth}(M).$$

In fact, Stanley [11] conjectured that

Stanley depth conjecture. Every $\mathbb{Z}^n$-graded S-module satisfies Stanley's inequality.

This conjecture has been recently disproved in [2]. In this talk, we describe their counterexample. Time permitting, We will also present the recent developments in this topic.

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