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Covering properties defined by stars

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Abstract

We discuss covering properties in topological spaces defined by stars. Special attention is paid to two star covering properties related to the Gerlits-Nagy property GN. Some examples in this connection are given.

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1 Introduction

If A is a subset of a topological space X, and $\mathcal{P}$ is a family of subsets of X, then $\mathrm{St}(A, \mathcal{P}) := \cup\{P \in \mathcal{P} : A \cap P \neq \emptyset\}$; when $A = \{x\}$, $x \in X$, one writes $\mathrm{St}(x, \mathcal{P})$ instead of $\mathrm{St}(\{x\}, \mathcal{P})$. In the literature one can find a big number of topological properties which are defined or characterized in terms of stars. In particular, it is the case with many covering properties of topological spaces. We consider here an application of this method in the theory of star selection principles introduced in [4]. For more details on star selection principles and for undefined notions see the survey paper [5].

Selection Principles Theory has roots in the papers by Menger [7], Hurewicz [3], Rothberger [9], but in the last two-three decades a big number of mathematicians work systematically in this field of mathematics.

Following [4] and [5] we have the following definitions.

Let $\mathcal{O}$ be the collection of all open covers of a space X, $\mathcal{B}$ a subfamily of $\mathcal{O}$, and $\mathcal{K}$ a family of subsets of X. Then:

1. The symbol $\mathsf{S}^*_{fin}(\mathcal{O}, \mathcal{B})$ denotes the selection hypothesis: For each sequence $\langle \mathcal{U}_n : n \in \mathbb{N}\rangle$ of elements of $\mathcal{O}$ there is a sequence $\langle \mathcal{V}_n : n \in \mathbb{N}\rangle$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$, and $\{\mathrm{St}(\cup\mathcal{V}_n, \mathcal{U}_n) : n \in \mathbb{N}\} \in \mathcal{B}$;
2. $\mathsf{S}^*_1(\mathcal{O}, \mathcal{B})$ denotes the selection hypothesis: For each sequence $\langle \mathcal{U}_n : n \in \mathbb{N}\rangle$ of elements of $\mathcal{O}$ there is a sequence $\langle U_n : n \in \mathbb{N}\rangle$ such that for each $n \in \mathbb{N}$, $U_n \in \mathcal{U}_n$ and $\{\mathrm{St}(U_n, \mathcal{U}_n) : n \in \mathbb{N}\} \in \mathcal{B}$;
3. $\mathsf{SS}^*_{\mathcal{K}}(\mathcal{O}, \mathcal{B})$ denotes the following selection hypothesis: For each sequence $\langle \mathcal{U}_n : n \in \mathbb{N}\rangle$ of elements of $\mathcal{O}$ there exists a sequence $\langle K_n : n \in \mathbb{N}\rangle$ of elements of $\mathcal{K}$ such that $\{\mathrm{St}(K_n, \mathcal{U}_n) : n \in \mathbb{N}\} \in \mathcal{B}$.
When $\mathcal{K}$ is the collection of all finite (resp. one-point, compact) subspaces of X we write $\mathsf{SS}^*_{fin}(\mathcal{O}, \mathcal{B})$ (resp., $\mathsf{SS}^*_1(\mathcal{O}, \mathcal{B})$, $\mathsf{SS}^*_{\mathrm{K}}(\mathcal{O}, \mathcal{B})$) instead of $\mathsf{SS}^*_{\mathcal{K}}(\mathcal{O}, \mathcal{B})$.