Domination number of the order graph of a group

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Abstract

The order graph of a group G, denoted by $\Gamma^*(G)$, is a graph whose vertices are non-trivial subgroups of G and two distinct vertices H and K are adjacent if and only if $|H|\big||K|$ or $|K|\big||H|$. In this paper, we study the domination number of this graph.

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1 Introduction

Let G be a finite group. The order graph of G is the (undirected) graph $\Gamma^*(G)$, whose vertices are non-trivial proper subgroups of G and two distinct vertices H and K are adjacent if and only if either $|H|\big||K|$ or $|K|\big||H|$. So $\Gamma^*(G)$ is the empty graph if and only if $|G|$ is a prime number. This graph has studied in [8] and [4]. In this paper, we study the domination number of this graph.

First we recall some facts and notations related to this paper. Throughout this paper G denotes a nontrivial finite group. Let $\pi(n)$ be the set of prime divisors of n. We denote $\pi(|G|)$ by $\pi(G)$. The cyclic group of order n is denoted by C_n. The symmetric group on n letters is denoted by S_n. D_n is the dihedral group of order $2n$. The alternative group is denoted by A_n. The finite field with q elements is denoted by $\mathbb{F}_q$.

Let Γ be a simple graph with vertex set V. A subset S of V is called a dominating set if every vertex in $V\backslash S$ has a neighbor in S. The minimum size of the dominating sets is called domination number and is denoted by $\gamma(\Gamma)$. We denote $\gamma(\Gamma^*(G))$ by $\gamma(G)$.

2 Main results

In this section we state and prove our main results.

Theorem 2.1. *Let S be a set of subgroups of G such that for each prime $p \in \pi(G)$ there is only one subgroup P of order p in S. Then S is a dominating set.*

Proof. Let H be a subgroup of G. Let p be a prime factor of $|H|$. If P is a subgroup of order p in S then $H = P$ or H is adjacent to P. □

Corollary 2.2. *The domination number of the order graph of G is at most $|\pi(G)|$.*

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