On the fixed point theorem for $\mathbf{C}^*$-algebra-valued 2-metric spaces

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Abstract

In this paper we first establish the structure of C^*-algebra-valued 2-metric space and then we give some fixed point theorems for self-maps with contractive or expansive conditions on such spaces.

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1 Introduction

The notion of C^*-algebra-valued metric spaces has been investigated by Z. Ma, L. Jiang and H. Sun [4]. They presented some fixed point theorems for self-maps with contractive or expansive conditions on such spaces. Very recently, the authors [1, 3] proved some fixed point theorems by introducing the notion of 2-metric spaces. Using the concepts of 2-metric spaces and C^*-algebra-valued metric spaces, we define a new type of extended metric spaces. Then, we prove some fixed point theorems in this structure.

We provide some notations, definitions and auxiliary facts which will be used later in this paper.
Let $\mathbb{A}$ be a unital algebra with unit I. An involution on $\mathbb{A}$ is a conjugate-linear map $a \mapsto a^*$ on $\mathbb{A}$, such that $a^{**} = a$ and $(ab)^* = b^*a^*$ for all $a, b \in \mathbb{A}$. An assign to each $*$-algebra is $(\mathbb{A}, *)$. A Banach $*$-algebra is a $*$-algebra $\mathbb{A}$ together with a complete submultiplicative norm such that $\|a^*\| = \|a\|$ for all $a \in \mathbb{A}$. A C^*-algebra is a Banach $*$-algebra such that $\|a^*a\| = \|a\|^2$ $(a \in \mathbb{A})$. For more details we refer the reader to [2].
Throughout this manuscript, $\mathbb{A}$ stands for a unital C^*-algebra with unit I. We say an element $x \in \mathbb{A}$ a positive element, denote it by $x \succeq \theta$, if $x = x^*$ and $\sigma(x) \subseteq \mathbb{R}_+ = [0, \infty)$, where θ means the zero element in $\mathbb{A}$ and $\sigma(x)$ is the spectrum of x. Using positive elements, one can define a partial ordering $\preceq$ as follows: $x \preceq y$ if and only if $y - x \succeq \theta$ $(x, y \in \mathbb{A})$. From now on, by $\mathbb{A}_+$ we denote the set $\{x \in \mathbb{A} : x \succeq \theta\}$ and $|x| = (x^*x)^{\frac{1}{2}}$.

Definition 1.1. ([4]) Let X be a nonempty set. Suppose the mapping $d : X \times X \to \mathbb{A}$ satisfies:
1) $\theta \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;
2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

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