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Ternary (σ, τ, ξ)-Derivations on Banach Ternary Algebras

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Abstract

Let A be a Banach ternary algebra over a scalar field $\mathbb{R}$ or $\mathbb{C}$ and X be a Banach ternary A-module. Let σ, τ and ξ be linear mappings on A. We define a ternary (σ, τ, ξ)-derivation and a Lie ternary (σ, τ, ξ)-derivation. Moreover, we prove the generalized Hyers-Ulam-Rassias stability of ternary and lie ternary (σ, τ, ξ)-derivations on Banach ternary algebras.

Keywords: Banach ternary A-module, Ternary (σ, τ, ξ)-derivation, Hyers–Ulam–Rassias stability.
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1 Introduction

Ternary algebraic operations were considered in the 19 th century by several mathematicians such as A. Cayley [3] who introduced the notion of cubic matrix which in turn was generalized by Kapranov, Gelfand and Zelevinskii in 1990 ([4]).
A ternary (associative) algebra $(A, [\ ])$ is a linear space A over a scalar field $\mathbb{F} = (\mathbb{R}$ or $\mathbb{C})$ equipped with a linear mapping, the so-called ternary product, $[\ ]\colon A \times A \times A \to A$ such that $[[abc]de] = [a[bcd]e]$ for all $a, b, c, d, e \in A$. This notion is a natural generalization of the binary case. It is known that unital ternary algebras are trivial and finitely generated ternary algebras are ternary subalgebras of trivial ternary algebras [1].

By a Banach ternary algebra we mean a ternary algebra equipped with a complete norm $\|.\|$ such that $\|[abc]\| \leq \|a\|\|b\|\|c\|$.

Let A be a Banach ternary algebra and X be a Banach space. Then X is called a ternary Banach A-module, if module operations $A \times A \times X \to X$, $A \times X \times A \to X$, and $X \times A \times A \to X$ are $\mathbb{C}$-linear in every variable. Moreover satisfy:

$$\max\{\|[xab]_X\|, \|[axb]_X\|, \|[abx]_X\|\} \leq \|a\|\|b\|\|x\|$$

for all $x \in X$ and all $a, b \in A$.

Let σ, τ and ξ be linear mappings on A. A linear mapping $D : (A, [\ ]_A) \to (X, [\ ]_X)$ is called a ternary (σ, τ, ξ)-derivation, if

$$D([abc]_A) = [D(a)\tau(b)\xi(c)]_X + [\sigma(a)D(b)\xi(c)]_X + [\sigma(a)\tau(b)D(c)]_X \tag{1}$$

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