



## The spectra of endomorphisms of analytic Lipschitz algebras

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### Abstract

In this paper the spectra of certain endomorphisms of the analytic Lipschitz algebras  $Lip_A(\bar{\mathbb{D}}, \alpha)$  are determined. We consider endomorphisms  $T$  of  $Lip_A(\bar{\mathbb{D}}, \alpha)$  defined by  $T(f) = f \circ \varphi$  for some  $\varphi \in Lip_A(\bar{\mathbb{D}}, \alpha)$  for the case where  $\varphi$  has an interior fixed point.

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## 1 Introduction

An endomorphism of an algebra  $B$  is a linear operator  $T$  of  $B$  into itself satisfying  $T(ab) = (Ta)(Tb)$  for all  $a, b \in B$ . If a Banach function algebra  $B$  on a compact Hausdorff space  $X$  is natural, then every nonzero endomorphism  $T$  of  $B$  has the form  $Tf = f \circ \varphi$  for a self-map  $\varphi$  of  $X$ . We call  $T$  the endomorphism of  $B$  induced by  $\varphi$ . The spectrum of an operator  $T$  on an algebra  $B$  is the set of complex numbers  $\lambda$  for which  $\lambda - T$  is not invertible. We denote the spectrum of an operator  $T$  by  $\sigma(T)$ .

Let  $(X, d)$  be a metric space and  $0 < \alpha \leq 1$ . The complex valued function  $f$  on  $X$  is said to satisfy the Lipschitz condition of order  $\alpha$  on  $X$ , if there exists a constant  $K > 0$  such that  $|f(x) - f(y)| \leq Kd(x, y)^\alpha$ , for all  $x, y \in X$ . In this case we write

$$p_\alpha(f) = \sup\left\{\frac{|f(x) - f(y)|}{d(x, y)^\alpha} : x, y \in X, x \neq y\right\}.$$

Suppose that  $\mathbb{D}$  is the open unit disc in the complex plane  $\mathbb{C}$ . The analytic Lipschitz algebra on the closed unit disc  $\bar{\mathbb{D}}$ ,  $Lip_A(\bar{\mathbb{D}}, \alpha)$  is the algebra of functions  $f$  analytic in the open unit disc  $\mathbb{D}$  that satisfy a Lipschitz condition of order  $\alpha$  on  $\bar{\mathbb{D}}$ . It is well known that the analytic Lipschitz algebra  $Lip_A(\bar{\mathbb{D}}, \alpha)$  is a natural Banach function algebra with the norm

$$\|f\| = |f|_{\bar{\mathbb{D}}} + p_\alpha(f) \quad (f \in Lip_A(\bar{\mathbb{D}}, \alpha)),$$

where  $|f|_{\bar{\mathbb{D}}} = \sup_{z \in \bar{\mathbb{D}}} |f(z)|$ .

Kamowitz in [2] determined the spectra of a class of endomorphisms of the disc algebra  $A(\bar{\mathbb{D}})$ , the uniform algebra of functions analytic on the open unit disc  $\mathbb{D}$  and continuous on

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