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Talk Directionally uniform distributions and their applications pp.: 1–4

Directionally Uniform Distributions and their applications

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Abstract

One of the main properties of the Gaussian distribution is the existence of a multivariate version which its directional marginals give Gaussian distributions with prescribed covariance. In this article we study the same property for uniform distribution. We formulate the concept of directionally uniform distributions and then prove that in dimensions 2 and 3 such distribution exist but in dimensions greater than 3 it does not exist.

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1 Introduction

Among continuous distributions, the normal distribution is probably the most interesting because of its several useful properties. One of its properties is the existence of the multivariate Gaussian distribution, which all of its linear combinations are Gaussian. This property make the Gaussian distribution computationally efficient and can be used to generate families of normal variables with prescribed mean and covariance.

One could ask if such multivariate version exists for other continuous distributions. In this article we study this problem for uniform distribution.

In section 2 we define the directionally uniform distribution in $\mathbb{R}^n$. In Theorem 2.2 and 2.4 we prove that this distribution exists only in 2 and 3 dimensions.

We also provide an application of the directionally uniform distribution in $\mathbb{R}^3$.

2 Main Results

2.1 Definition

Definition 2.1. By an n dimensional directionally uniform distribution we mean a probability measure on $\mathbb{R}^n$ with the property that its projection on any direction is a uniform distribution on an interval.

The first problem is the existence of such distributions. We will show that in dimensions 2 and 3 it exists but for $n \geq 4$ it does not exist.