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Some quotient graphs of the power graphs*

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Abstract

In this paper we define three quotient graphs of the power graphs and study their properties and some relation between them.

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1 Introduction

Let G be a finite group. The *power graph* $P(G)$ is the graph with vertex set G and edge set E, where there is an edge $\{x, y\} \in E$ between two distinct vertices $x, y \in G$ if one is a positive power of the other (see [2]). Observed that $P(G)$ is 2-connected if and only if $P_0(G)$, the 1_G-cut subgraph of $P(G)$, is connected. Many of results are collected in a survey [1].

In this paper we define quotient power graph, order graph and power type graph of a finite group and study some properties of them, particulary the 2-connectivity of them. Throughout this paper, we use the standard notations of [4]. Also we denote by $c(\Gamma)$, the number of connected components of the graph Γ.

Definition 1.1. Let $\Gamma = (V, E)$ be a graph and $\sim$ is an equivalence relation on the set V. The *quotient graph* $\Gamma/\sim = ([V], [E])$, of Γ with respect to $\sim$ is a graph with vertex set $[V] = V/\sim$ and there is an edge $\{[x], [y]\} \in [E]$ between $[x], [y] \in [V]$ if $[x] \neq [y]$ and there exist $\overline{x}, \overline{y} \in V$ such that $\overline{x} \sim x$, $\overline{y} \sim y$ and $\{\overline{x}, \overline{y}\} \in E$.

Definition 1.2. Define the equivalence relation relation $\sim$ on G as follows: For $x, y \in G$, $x \sim y$ if and only if $\langle x \rangle = \langle y \rangle$. Then $[x] = \{x^m : 1 \leq m \leq o(x), \ (m, o(x)) = 1\}$. The quotient graph $P(G)/\sim = ([G] = G/\sim, [E])$ will be denoted by $\widetilde{P}(G)$ and called the *quotient power graph* of G. We show that $[x] \neq [y]$, $\{[x], [y]\} \in [E]$ if and only if $\{x, y\} \in E$. $\widetilde{P}(G)$ is always connected and it is 2-connected if and only if the 1_G-cut subgraph $\widetilde{P}_0(G)$, of $\widetilde{P}(G)$, is connected.

Definition 1.3. The *order graph* of G is the graph $\mathcal{O}(G)$ with vertex set $O(G) = \{m \in \mathbb{N} : \exists g \in G \text{ with } o(g) = m\}$ and edge set $E_{\mathcal{O}(G)}$, where for each $m, n \in O(G)$, $\{m, n\} \in E_{\mathcal{O}(G)}$ if $m \neq n$ and $m \mid n$ or $n \mid m$. The *proper order graph* $\mathcal{O}_0(G)$ is defined as the 1-cut graph of $\mathcal{O}(G)$. Its vertex set is then $O_0(G) = O(G) \setminus \{1\}$. We set $c(\mathcal{O}_0(G)) = c_0(\mathcal{O}(G))$. $\mathcal{O}(G)$ is always connected and it is 2-connected if and only if $\mathcal{O}_0(G)$ is connected.

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