A covering property of pullback fibrations

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Abstract

In this talk, we show that the total space of the pullback fibration of a given fibration $f : E \longrightarrow X$ by a covering map $c : \widetilde{X} \longrightarrow X$ is a covering space of E. Also, we study conditions in which give us universal property of these covering maps.

Keywords: Covering map, Fibration, Pullback fibration
Mathematics Subject Classification [2010]: 57M10, 55R05

1 Introduction

We recall that a *fibration* $f : E \longrightarrow X$ is a continuous map in which has homotopy lifting property with respect to an arbitrary space. If $f : E \longrightarrow X$ is a fibration and $g : Y \longrightarrow X$ is a continuous map, then

$$g^*E := \{(y, e) \in Y \times E | g(y) = f(e)\},$$

equipped by the subspace topology and projection map $p_1 := pr_1 : g^*E \longrightarrow Y$ defines a fibration over Y, named *pullback* of f by g and denoted by g^*f. Also, recall that a *covering map* is a continuous map $c : \widetilde{X} \longrightarrow X$ such that for every $x \in X$ there exists an open subset U of X with $x \in U$ for which U is *evenly covered* by c, that is, $c^{-1}(U)$ is a disjoint union of open subsets of $\widetilde{X}$ each of which is mapped homeomorphically onto U by c. When $\widetilde{X}$ is simply connected, c is called universal covering and it is called categorical universal covering if for every covering map $q : \widetilde{Y} \longrightarrow X$ with a path connected total space $\widetilde{Y}$, there exists a covering map $f : \widetilde{X} \longrightarrow \widetilde{Y}$ such that $q \circ f = c$.

In this note, we consider the following commutative diagram:

$$\begin{tikzcd} c^*E \arrow[r, "p_2"] \arrow[d, "p_1"'] & E \arrow[d, "f"] \\ \widetilde{X} \arrow[r, "c"] & X, \end{tikzcd}$$

where c is covering map, f is fibration and p_1 is pullback fibration of f by c.

It is well-known that p_1 is a fibration [4, page 98]. Here we prove that if c is a covering map, then p_2 is also a covering map and hence we have a diagram such that vertical maps are fibration and horizontal maps are covering map. Next, we show that if c is universal covering map, p_2 is not necessarily universal unless f has unique path lifting property.

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