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Some new families of 2-regular self-complementary k-hypergraphs for $k = 4, 5$

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Abstract

A k-hypergraph with vertex set V and edge set E is called t-regular if every t-element subset of V lies in the same number of elements of E. In this note, we prove the existence of some new families of 2-regular self-complementary k-hypergraphs for k=4,5.

Keywords: k-hypergraph, self-complementary hypergraph, large sets of t-designs
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1 Introduction

A *k-uniform hypergraph* of order v is an ordered pair $H = (V, E)$, where $V = V(H)$ is a v-set (called *vertex set*) and $E = E(H)$ (called *edge set*) is a subset of the set of all k-subsets of V ($P_k(V)$). We call a *k-uniform hypergraph* simply a *k-hypergraph* [4]. A k-hypergraph H of order v is *t-subset-regular* (for short *t-regular*) if there exists a positive integer λ (called the *t-valence* of H), such that each element of $P_t(V)$ is a subset of exactly λ elements of $E(H)$. Henceforth, we denote such a structure by RHG(t, k, v). Two k-hypergraphs H_1 and H_2 are isomorphic, if there is a bijection $\theta : V(H_1) \to V(H_2)$, such that θ induces a bijection from $E(E_1)$ into $E(H_2)$. A k-hypergraph H is called *self-complementary* if H is isomorphic to $H' = (V, P_k(V) \setminus E(H))$. An *antimorphism* of self complementary hypergraph H, is an isomorphism between H and H'. Henceforth, we denote this structure by SRHG(t, k, v). An easy counting argument shows that an SRHG(t, k, v) is also an SRHG(i, k, v) for $0 \le i \le t$. Hence a set of necessary conditions for the existence of an SRHG(t, k, v) is that $\binom{v-i}{k-i}$ is an even integer for all $i = 0, 1, ..., t$. The following theorem gives the necessary conditions in terms of some congruence relations. Let p be a prime number and r and m be positive integers. Then by $r_{[m]}$ we denote the remainder of division r by m and by $r_{(p)}$ we denote the largest integer i such that p^i divides r.

Theorem 1.1. *[2] If there exists an SRHG(t, k, v), then there exists an integer q, where $k_{(2)} < q \le \min\{i \;:\; 2^i \;>\; k\}$ such that $v_{[2^q]} \in \{t, t+1, ..., k_{[2^q]} - 1\}$.*

It should be noted that in [2] the above theorem is stated for large sets of t-designs. We may obtain more hypergraphs from a given hypergraph as the following theorem suggests (see [4]). The proof is clear by successive applying of the above remark.

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