46th Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University
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# Some iterative methods for solving an operator equation by using g-frames

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**Abstract**

This paper proposes some iterative methods for solving an operator equation on a separable Hilbert space $H$ equipped with a g-frame. We design some algorithms based on the Richardson and Chebyshev methods and investigate the convergence and optimality of them.
**Keywords:** Hilbert space, g-frame, operator equation, iterative method, Chebyshev polynomials.
**MSC(2010):** Primary: 47J25; Secondary: 46C05.

## 1 Introduction and preliminaries

G-frames are natural generalization of frames and provide more choices on analyzing functions from frame expansion coefficients. Let $J$ be a countable index set and $\{\Lambda_j\}_{j\in J}$ be a set of operators from a separable Hilbert space $H$ to another separable Hilbert space $V_j$ for $j \in J$. The sequence $\{\Lambda_j\}_{j\in J}$ is called a $g-frame$ for $H$ with respect to $\{V_j\}_{j\in J}$ if there are two positive $A$ and $B$ such that

$$A\|f\|^2 \leq \sum_{j\in J} \|\Lambda_j f\|^2 \leq B\|f\|^2, \qquad \forall f \in H.$$

$A$ and $B$ is called the lower and upper frame bound, respectively. If $A = B$ then $\{\Lambda_j\}_{j\in J}$ is called a tight g-frame. The g-frame operator $S$ for a g-frame $\{\Lambda_j\}_{j\in J}$, for $H$ with respect to $\{V_j\}_{j\in J}$, is defined by

$$Sf = \sum_{j\in J} \Lambda_j^* \Lambda_j f, \qquad \forall f \in H,$$

where $\Lambda_j^*$ is the adjoint operator of $\Lambda_j$.
It is easy to check that $S$ is a bounded, invertible and self-adjoint operator and

$$AI \leq S \leq BI \ \ , \ \ \frac{1}{B}I \leq S^{-1} \leq \frac{1}{A}I.$$

Writing $\tilde{\Lambda_j} = \Lambda_j S^{-1}$, then for any $f \in H$ we have

$$f = \sum_{j\in J} \Lambda_j^* \tilde{\Lambda_j} f = \sum_{j\in J} \tilde{\Lambda_j}^* \Lambda_j f.$$

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