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Steklov problem for a three-dimensional Helmholtz equation in bounded domain

Mojtaba Sajjadmanesh*
University of Bonab

Parisa Vafadar
Teacher at Education Department of Bonab

Abstract

This paper is devoted to study of solutions of a Steklov problem for a three-dimensional Helmholtz equation with an eigenvalue parameter λ in the non-local boundary conditions on the two-party smooth boundary of a connected bounded domain.The derived necessary conditions construct a system of second kind Fredholm integral equations with multi-dimensional singular integrals. Finally,a new method for regularization of these singularities is represented.

Keywords: Steklov problem, Fundamental solution, Fredholm integral equation
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1 Introduction

Our method for investigation of these problems has been used for the first and second order elliptic equations such as Cauchy-Riemann and Laplace equations with non-local boundary conditions in a two-dimensional bounded domain [1], [2] and in this paper, we apply this process for a three-dimensional elliptic equation.

2 Problem statement

Let Ω be a simply connected bounded domain in $\mathbb{R}^3$ and its boundary Γ is a Lyapunov surface which contains two parts;
$\Gamma = \Gamma_1 \cup \Gamma_2,\ \Gamma_1 :\ x_3 = \gamma_1(x')\ ,\ \Gamma_2 :\ x_3 = \gamma_2(x')\ ; (\gamma_2(x') < \gamma_1(x'))\ \ x' \in S$,
where S is the projection of the domain Ω on the plane Ox_1x_2.
Let's consider Helmholtz equation

$$Lu(x) = (\Delta + k^2)u(x) = 0 \qquad in \quad \Omega \subset \mathbb{R}^3, \tag{1}$$

with non-local boundary conditions:

$$\sum_{k=1}^{3}[\alpha_{jk}(x')\frac{\partial u(x)}{\partial x_k}|_{x_3=\gamma_1(x')} + \beta_{jk}(x')\frac{\partial u(x)}{\partial x_k}|_{x_3=\gamma_2(x')}] = \lambda u(x', \gamma_j(x')),\ j = 1, 2 \tag{2}$$

The coefficients $\alpha_{jk}, \beta_{jk},\ j = 1, 2, k = 1, 2, 3$ are known C^1 functions and λ is a spectral parameter.

*Speaker