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Talk On topologies generated by subrings of the algebra of all real-valued functions pp.: 1–4

On topologies generated by subrings of the algebra of all real-valued functions

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Abstract

Let X be a topological space and R be a subring of $\mathbb{R}^X$. Associated with the subring R, we generalize the separation axioms on X. Moreover, we specify three topologies on X, namely $Z(R)$-topology, $Coz(R)$-topology and the weak topology induced by R. Comparsion and coincidence of each pair of these topologies are investigated. Using these topologies, a one-one correspondence between points of X and fixed maximal ideals of R is given

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1 Introduction

Throughout this article, $\mathbb{R}^X$ denotes the algebra of all real-valued functions on X and $C(X)$ (resp., $C^*(X)$) denotes the subalgebra of $\mathbb{R}^X$ consisting of all continuous functions (resp., bounded continuous functions). Note that X is not necessarily a Tychonoff space. For each $f \in \mathbb{R}^X$, $Z(f) = \{x \in X : f(x) = 0\}$ denotes the zero-set of f and $Coz(f)$ denotes the complement of $Z(f)$ with respect to X. For a subring R of $\mathbb{R}^X$, $Z(R)$ denotes $\{Z(f) : f \in R\}$, cleary $Z(C(X)) = Z(X) = \{Z(f) : f \in C(X)\}$. Also, we use $M_x(R)$ to denote $\{f \in R, x \in Z(f)\}$. An ideal I in R is called free, if $\bigcap_{f\in I} Z(f) = \emptyset$. Otherwise, it is called fixed. By a maximal fixed ideal of R, we mean a fixed ideal that is maximal in the set of all fixed ideals of R. Clearly, fixed maximal ideals in $C(X)$ coincide with maximal fixed ideals and have the form $M_x = \{f \in C(X) : x \in Z(f)\}$, for $x \in X$. Note that for a subset A of X, M_A denotes $\{f \in C(X) : A \subseteq Z(f)\}$. The intersection of all the free ideals in $C(X)$ is denoted by $C_K(X)$. It is well-known that $C_K(X)$ is the subset of $C(X)$ consisting of all functions with compact support. Note that $cl_X Coz(f)$ is called the support of f for every $f \in C(X)$. The annihilator of $f \in R$ is defined by $Ann_R(f) = \{g \in R : fg = 0\}$. Assume that P and Q are partially ordered sets, then a function $f : P \longrightarrow Q$ is called an order-homomorphism if whenver $a \leq b$, then $f(a) \leq f(b)$. The function f is called an order-isomorphism if it is moreover bijective and $f^{-1} : Q \longrightarrow P$ is also an order-homomorphism. For terms and notations not defined here we follow the standard text of [4].

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