Integrating of Ordinary Differential Equations via a One-Parameter Symmetry Group

S. Reza Hejazi
University of Shahrood

Mustafa Bazghandi*
University of Shahrood

Sousan Abbaspour
University of Shahrood

Abstract

The main goal of this article is concerned with finding first integrals for first-order and higher-order ODEs and using one-parameter symmetries to reduce the order of a given ODE. All techniques, which are expressed in this article, are illustrated by examples.

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1 Introduction

There are many techniques for integrating of differential equations, but most works are only for very limited class of problems. Surprisingly, most well-known techniques have a common feature: they exploit symmetries of differential equations. The most basic type of symmetry is a group of point transformations acting on the space of independent and dependent variables. Lie's fundamental observation was that knowledge of a sufficiently large group of symmetries of a system of ODEs allow one to integrate the system by quadratures and thereby deduce the general solution. This approach unifies and significantly extends the various special methods introduced for the integration of ODEs. In this article, a survey of these methods is presented.

We begin the first section by a brief definition of a system of differential equations with the total space of dependent and independent variables, then the prolong formulation for finding the Lie algebra of symmetries including the invariance condition theoremare given. These two ones together construct a computational method for finding the Lie algebra of the symmetry group which is a vector space spanned by some vector fields called *infinitesimal generators* correspond to the transformations in the Lie group of symmetries. Finally in the main section we apply the symmetries to integrate a given ODE.

2 Mathematical Formulation

This section starts with a geometrical definition of a system of differential equations.

*Speaker