Some properties Sturm-Liouville problem with fractional derivative

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Abstract

In this paper we establish the properties of Fractional singular Sturm-Liouville problem. Our main issue is to investigate the spectral properties for the operator. Furthermore, we prove new results according to the fractional Sturm-Liouville problem.

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1 Introduction

We consider the following SturmLiouville problem with factional derivative in the leading term

$$\begin{cases} -{}^cD_{0^+}^{\alpha}u(t) + q(t)u(t) = \lambda u(t), & 0 < t < 1, \\ u(0) = u(1) = 0, \quad \alpha \in (1,2) \end{cases} \tag{1}$$

Definition 1.1. [2] (RiemannLiouville fractional integrals)We define the left and the RiemannLiouville fractional integrals by

$$I_{0^+}^{\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}f(s)ds,$$

where $\Gamma(\cdot)$ is the Euler gamma function.

Definition 1.2. [2] The Riemann-Liouville fractional derivative of order $\alpha > 0$, $n-1 < \alpha < n$, $n \in \mathbb{N}$ is defined as

$${}^cD_{0^+}^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{n-\alpha-1}f^{(n)}(s)ds,$$

where the function $f(t)$ have absolutely continuous derivatives up to order $(n-1)$.

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