Minimal description for the real interpolation in the case of quasi-Banach quaternion

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Abstract

We give a minimal description in the sense of Aronszajn-Gagliardo for the real methods in the case of quasi-Banach quaternion.

Keywords: quasi-Banach spaces, interpolation space, real method of interpolation
Mathematics Subject Classification [2010]: 46M35, 47A60

1 Introduction

Our main reference to the theory of interpolation space is [1]. Let $\bar{A} = (A_0, A_1, A_2, A_3)$ be a quasi-Banach quaternion and $\bar{t} = (t_1, t_2, t_3) \in R^3_+$. The Peetre' K-functional is defined for $a \in A_0 + A_1 + A_2 + A_3 := \sum(\bar{A})$ by
$K(t_1, t_2, t_3, a; \bar{A})$

$$= \inf\{\|a_0\|_{A_0} + t_1\|a_1\|_{A_1} + t_2\|a_2\|_{A_2} + t_3\|a_3\|_{A_3} \ : \ a = \sum_{i=0}^{3} a_i, \ a_j \in A_j\}$$

and similarly the J-functional for $a \in A_0 \cap A_1 \cap A_2 \cap A_3 := \triangle(\bar{A})$ by

$$J(t_1, t_2, t_3, a; \bar{A}) = max\{\|a\|_{A_0}, t_1\|a\|_{A_1}, t_2\|a\|_{A_2}, t_3\|a\|_{A_3} \ : \ a \in \triangle(\bar{A})\}.$$

Let $\bar{A} = (A_0, A_1, A_2, A_3)$ be a quaternion of quasi-Banach spaces and $\bar{n} = (n_1, n_2, n_3) \in Z^3$. For $0 < \theta_1, \theta_2, \theta_3 < 1, \theta_1 + \theta_2 + \theta_3 < 1$ and $0 < q \leq \infty$ we define the real interpolation space $\bar{A}_{(\theta_1,\theta_2,\theta_3),q,K}$ as the set of all $a \in \sum(\bar{A})$ which have a finite quasi-norm
$\|a\|_{(\theta_1,\theta_2,\theta_3),q,K}$

$$= \begin{cases} \left(\sum_{\bar{n} \in Z^3} (2^{-n_1\theta_1} 2^{-n_2\theta_2} 2^{-n_3\theta_3} K(2^{n_1}, 2^{n_2}, 2^{n_3}, a; \bar{A}))^q \right)^{1/q} & if \ 0 < q < \infty \\ \sup_{\bar{n} \in Z^3} \{2^{-n_1\theta_1} 2^{-n_2\theta_2} 2^{-n_3\theta_3} K(2^{n_1}, 2^{n_2}, 2^{n_3}, a; \bar{A})\} & if \ q = \infty \end{cases}.$$

Also we define the real interpolation space $\bar{A}_{(\theta_1,\theta_2,\theta_3),q,J}$ as the set of all $a \in \sum(\bar{A})$that may by written as $a = \sum_{\bar{n} \in Z^3} u_{\bar{n}}, \ u_{\bar{n}} \in \triangle(\bar{A})$ (convergence in $\sum(\bar{A})$) and which have a finite

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