Upper bound for the number of limit cycles in a Lienard system

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Abstract

Lienard system forms one of the important class of differential equations which is considered widely in recent years. An interesting problem studied about this equations is to obtain an upper bound for the number of limit cycle. In this paper we study hopf bifurcation for special polynomial Lienard system and find a maximal number of limit cycle near the origin which named Hopf cyclicity.

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1 Introduction

Consider the Lienard equation $\ddot{x} + f(x)\dot{x} + g(x) = 0$, which has a equivalent form

$$\dot{x} = y - F(x), \qquad \dot{y} = -g(x) \tag{1}$$

where $F(x) = \int_0^x f(x)dx$. Depending of F and g, this system has been widely studied by mathematicans and scientists and many conclusions about the number of limit cycles for this system are obtained. For example in [1] it is proved that the system

$$\dot{x} = y - \frac{\Sigma_{i=0}^{n} a_i x^i}{1 + \Sigma_{i=1}^{m} b_i x^i}, \qquad \dot{y} = -g(x)$$

has Hopf cyclicity $[\frac{n+m-1}{2}]$ at the point $(0, a_0)$, where $g(-x) = -g(x)$, $g(0) = 0$ and $g\prime(0) > 0$. In [2] author gives the number $[\frac{4n+2m-4}{3}] - [\frac{n-m}{3}]$ ($n \geqslant m$), as an upper bound for the maximum number of limit cycle in neighborhod of the point $(0, a_0)$, for above system with $gx) = x(x+1)$, where this number is Hopf cyclicity in case $n = m$. Now suppose F and g be polynomials. Let $\widehat{H}(i,j)$ denote the maximal number of small-amplitude limit cycle bifurcated from a focus of system (1), where i and j are degree of f and g. Yu and Han in [3] gave a table on $\widehat{H}(i,j)$ which summarizes the existing result for some i and j. In particular if g is quadratic then $\widehat{H}(i,2) = [\frac{2i+1}{3}]$ for $i \geqslant 2$, and if g is cubic then $\widehat{H}(i,3) = 2[\frac{3i+6}{8}]$ for $2 \leqslant i \leqslant 50$. Yu and Lynch in [4] considered two type of symmetric Lienard systems and proved that the system

$$\dot{x} = y - \Sigma_{i=0}^{m} a_i x^{2i+1}, \quad \dot{y} = -x(x^2 - 1)$$

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