On Generalized Covering Subgroups of a Fundamental Group

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Abstract

In this talk, after reviewing concepts of covering, semicovering and generalized covering subgroups introduced by J. Brazas, we give a new criterion for a subgroup $H \leq \pi_1(X, x_0)$ to be a generalized covering subgroup.

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1 Introduction

Recently, the notion of covering space has been extended using eliminating some of its properties and keeping some others [1,2,3,5]. For instance, semicoverings are introduced by eliminating the evenly covered property and keeping local homeomorphismness and unique path lifting property [2]. In the case of generalized coverings, local homeomorphismness has been replaced with unique lifting property [1,3,5]. It is well-khown that for connected and locally path connected spaces every covering is a semicovering and every semicovering is a generalized covering. Let $p : (\tilde{X}, \tilde{x_0}) \to (X, x_0)$ be a map and $H = p_*\pi_1(\tilde{X}, \tilde{x}_0) \leq \pi_1(X, x_0)$. Then H is called a covering, a semicovering or a generalized covering subgroup if p is covering, semicovering or generalized covering map, respectively. It is shown that H is a covering subgroup if and only if it contains an open normal subgroup of $\pi_1^{qtop}(X, x_0)$ [2,6]. Brazas showed that H is a semicovering subgroup if and only if it is an open subgroup of $\pi_1^{qtop}(X, x_0)$. He also proved that H is a generalized covering subgroup if and only if $p_H : \tilde{X}_H \to X$ has the uniqe path lifting property, where $p_H : \tilde{X}_H \to X$ is the well-known endpoint projection [3]. Now in this talk, we show that for a connected and locally path connected space X, a subgroup H of $\pi_1(X, x_0)$ is a generalized covering subgroup if and only if $(p_H)_*\pi_1\left(\tilde{X}_H, e_H\right) = H$.

2 Notations and Preliminaries

Definition 2.1. A pointed continuous map $p : (\tilde{X}, \tilde{x_0}) \to (X, x_0)$ has **UL (unique lifting)** property if for every connected, locally path connected space (Y, y_0) and every continuous map $f : (Y, y_0) \longrightarrow (X, x_0)$ with $f_*\pi_1(Y, y_0) \subseteq p_*\pi_1(\tilde{X}, \tilde{x}_0)$, there exists a

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