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Talk Positive solutions of nonlinear fractional differential inclusions pp.: 1–4

Positive solutions of nonlinear fractional differential inclusions

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Abstract

In this paper, we study fractional differential inclusions with integral boundary value conditions. We prove the existence of a solution under both convexity and nonconvexity conditions on the multi-valued right-hand side. The proofs rely on Bohnenblust-Karlin's fixed point theorem, and Covitz and Nadlers fixed point theorem for multivalued contractions.

Keywords: Fractional differential inclusions; Fractional derivative; Fractional integral; Fixed point
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1 Introduction

The purpose of this paper is to study a fractional differential inclusions with multi-point boundary conditions given by

$$\begin{cases} {}^cD^{\alpha}_{0^+}u(t) \in F(t,u(t)), & t \in (0,1),\ 2<\alpha<3, \\ u(0)=u''(0), \quad u(1)=\lambda\int_0^1 u(s)ds \end{cases} \tag{1}$$

where ${}^cD^{q}_{0^+}$ is the Caputo's fractional derivative, $2<\alpha<3$, and $0<\lambda<2$, $F:[0,1]\times\mathbb{R}\to P(\mathbb{R})$ is a multivalued map, $P(\mathbb{R})$ is the family of all subsets of $\mathbb{R}$.

We establish existence results for the problem (1), when the right-hand side is convex as well as non-convex valued. The first result relies on Bohnenblust-Karlin's fixed point theorem. In the second result, we shall use the fixed point theorem for contraction multivalued maps due to Covitz and Nadler.

In this section we sum up some basic facts that we are going to use later.

For a normed space $(X,\|\cdot\|)$, let

$P(X)=\{Y\subset X : Y\neq\emptyset\}$
$P_{cp}(X)=\{Y\in P(X) : Y \textit{ is compact}\}$
$P_{c}(X)=\{Y\in P(X) : Y \textit{ is convex}\}$
$P_{cl}(X)=\{Y\in P(X) : Y \textit{ is closed}\}$
$P_{b}(X)=\{Y\in P(X) : Y \textit{ is bounded}\}$
$P_{cp,c}(X)=\{Y\in P(X) : Y \textit{ is compact and convex}\}$

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