



Necessary and Sufficient Conditions for Weak Efficiency on K-subdifferentiable Functions

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Abstract

In this paper a multiobjective problem (MP) with a feasible set defined by inequality and equality constraints and a set constraint are considered. Then, by using the concept of K-directional derivatives, we obtain necessary and sufficient conditions for local weak efficiency on a new class of functions.

Keywords: K-directional derivative, Local cone approximation, Constraint qualification, Efficiency, Weak Efficiency.

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1 Introduction

The subject of optimality conditions for optimization problems, which are not necessarily smooth, has been extensively researched for the past decade. Central to this study has been the development of the appropriate cone approximations for generalizing the concepts of directional derivatives and subdifferentials. In this way, Elster and Thierfelder [4, 3] and independently Ward [7] exploiting a general and axiomatic definition of local cone approximation of a set, introduced a general definition of directional derivative for a function $f : X \rightarrow R$ where X is a finite dimensional space or also a topological linear space. Also, Nobakhtian [6] by using the concept of K-directional derivative proved general optimality conditions for a multiobjective problem with a feasible set defined by equality and inequality constraints. In this paper, we introduce a new class of functions and prove that under a suitable constraint qualification, it is a both necessary and sufficient condition in order to K-strongly efficient stationary points, K-weakly efficient stationary points, local efficient, local weak efficient, efficient and weak efficient be equivalent.

2 Notations and Preliminaries

Given the function $f : X \rightarrow R$, the epigraph of f is $\text{epi} f = \{(x, r) \in X \times R : f(x) \leq r\}$. The set $\text{epi} f$ will be locally approximated at the point $(x, f(x))$ by a local cone approximation K and a positively homogenous function $f^K(x, \cdot)$ will be uniquely determined.

Definition 2.1. ([2]) Let $f : X \rightarrow R, x \in X$ be a local cone approximation; the positively homogeneous function $f^K(x; \cdot) : R^\ell \rightarrow [-\infty, +\infty]$ defined by $f^K(x; d) := \inf\{\xi \in R : (d, \xi) \in K(\text{epi} f; (x, f(x)))\}$ is called the K-directional derivative of f at x .