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Cospectral Regular graphs

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Abstract

Graphs G and H are called cospectral if they have the same characteristic polynomial, equivalently, if they have the same eigenvalues considering multiplicities. Generalizing the construction of $G_4(a,b)$ and $G_5(a,b)$ due to Wang and Hao, we define graphs $G_4^r(a,b)$ and $G_5^r(a,b)$ and show that they are cospectral only if $r=1$ and $a+2=b$.

Keywords: eigenvalue, cospectral graphs, adjacency matrix, integral graphs.
Mathematics Subject Classification [2010]: 05C50

1 Introduction

We consider simple graphs, that is, graphs without loops or parallel edges. For basic notions in graph theory we refer to [4], whereas for preliminaries on graphs and matrices, see [1]. By the eigenvalues of a graph G, we mean the eigenvalues of its adjacency matrix $A(G)$. Graphs G and H are said to be cospectral if they have the same eigenvalues, counting multiplicities, or equivalently, they have the same characteristic polynomial. There is considerable literature on construction of cospectral graphs.

This paper is motivated by [3]. Bussemaker and Cvetković [2] introduced connected integral cubic graphs, denoted $G_1, G_2, \ldots, G_{13}$, among which G_4 and G_5 are cospectral. Wang and Hao [3] constructed graphs $G_4(a,b)$ and $G_5(a,b)$ based on G_4 and G_5. They showed that for any positive integer a, $G_4(a,a+2)$ and $G_5(a,a+2)$ form a pair of integral cospectral $(a+2)-$regular graphs, and concluded that there exist infinitely many pairs of cospectral integral graphs. We first give a generalization of $G_4(a,b)$ and $G_5(a,b)$ based on the method used in Lemma 1.1. We determine the characteristic polynomial of the resulting graphs. We also show that $G_4(a,b)$ and $G_5(a,b)$ are cospectral if and only if $a+2=b$.

Lemma 1.1. *Suppose that X and Y are square matrices of the same order. Let*

$$T = \begin{pmatrix} X & Y & \ldots & Y \\ Y & X & \ldots & Y \\ \vdots & \vdots & \ddots & \vdots \\ Y & Y & \ldots & X \end{pmatrix} \tag{1}$$

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