2-capability and 2-exterior center of a group

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Abstract

The aim of this talk is to obtain a characteristic subgroup of G to give a criteria for detecting 2-capability of G. We show that a relation between this subgroup and 2-epicenter of any group.

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1 Introduction and Motivation

The concept of epicenter $Z^*(G)$ is defined by Beyl and others in [1]. It gives a criteria for detecting capable groups. In fact G is capable if and only if $Z^*(G) = 1$. Ellis defined the exterior center $Z^\wedge(G)$ of G the set of all elements g of G for which $g \wedge h = 1$ for all $h \in G$ and he showed $Z^*(G) = Z^\wedge(G)$.

Similar to the concept of capability of group, a group G is called 2-capable if here exists a group H such that $G \cong H/Z_2(H)$. The concepts of $2-$capability and $2-$epicenter, $Z_2^*(G)$, were introduced by Ellis in [2]. Later Moghaddam and Kayvanfar in [4] showed that the $2-$epicenter $Z_2^*(G)$ of G is minimal subject to being the image of G of some $\mathcal{N}_2$ extensions of G, that is,

$$Z_2^*(G) = \bigcap_{(E,\phi)\text{is } \mathcal{N}_2 \text{ extension of } G} \phi(Z_2(E)).$$

Let G be a finite group presented as the quotient of a free group F by a normal subgroup R, following the notation in [2], we may define

$$\gamma_3^*(G) = \gamma_3(F)/\gamma_3(R,F) \text{ and } Z_2^*(G) = \pi(Z_2(F/\gamma_3(R,F)))$$

where $\pi : F/\gamma_3(R,F) \twoheadrightarrow G \cong F/R$ is an epimorphism given by $\gamma_3(R,F)x \mapsto Rx$.

Recall that the 2-nilpotent multiplier of G is the abelian group $\mathcal{M}^{(2)}(G) = \frac{R\cap\gamma_3(F)}{[R,F,F]}$, and the following sequence is exact

$$\mathcal{M}^2(G) \hookrightarrow \gamma_3^*(G) \twoheadrightarrow \gamma_3(G).$$

The main result of [2] shows G is $2-$capable if and only if $Z_2^*(G) = 1$.

It the current note, we define 2-exterior center $Z_2^\wedge(G)$ of G, and then we get that $Z_2^*(G) = Z_2^\wedge(G)$.

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