46th Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University

Some results on 2-modular spaces

Fatemeh Lael*
Department of Mathematics, Buein Zahra Technical University, Buein Zahra, Qazvin, Iran

Abstract

In this talk, first we review and discuss the concepts of 2-norm and 2-modular. Then, we prove that every 2-modular induces a 2-F-norm. In particular, we show that a β-homogeneous 2-modular induces a 2-F-norm with a special form.

Keywords: 2-modular, 2-modular spaces, 2-norm
Mathematics Subject Classification [2010]: 46A80

1 Introduction

A real valued function $\rho(\cdot,\cdot)$ on X^2 which X is a linear space, is said to be a 2-modular on X if it satisfies the following properties:

1. $\rho(x, y) = 0$ if and only if x, y are linearly dependent,
2. $\rho(x, y) = \rho(y, x)$,
3. $\rho(-x, y) = \rho(x, y)$,
4. $\rho(x, \alpha y+\beta z) \leq \rho(x, y)+\rho(x, z)$, for any nonnegative real numbers α, β with $\alpha+\beta = 1$.

The spaces equipped with two-modulars introduced by J. Musielak and A. Waszak [5] and generalized by K. Nourouzi and S. Shabanian [4]. In [1] and [2], Gahler developed the notion of a normed space to 2-normed spaces.

This work is devoted to study the relation between two-modular spaces and two-norm spaces.

Example 1.1. Let $X = \mathbb{R}^2$. Then

$$\rho(x_1, x_2) = \begin{cases} 1 & x_1, x_2 \text{ are linearly independent,} \\ 0 & x_1, x_2 \text{ are linearly dependent,} \end{cases}$$

is a 2-modular on X.

Definition 1.2. The set defined by

$$X_\rho = \{x \in X : for\ each\ y \in X,\ \rho(\lambda x, y) \to 0\ as\ \lambda \to 0\}$$

is called a 2-modular space.

*Speaker