Results for The Daugavet Property and Examples

L. Nasiri *
University of Lorestan

M. Shakoori
University of Lorestan

Abstract

In present paper, results of the Daugavet property for Banach spaces. Also express several examples that show in general the Daugavet property is not transmitted from space into subspace and vice-versa.

Keywords: The Daugavet property ,Banach space, Almost narrow and narrow operators
Mathematics Subject Classification [2010]: 13D45, 39B42

1 Introduction

In this section we state several results and examples for Banach spaces with the Daugavet property.
Definition 1.1 Banach space X has the Daugavet property if every rank-1 operator $T \in L(X)$ satisfies (1).
Definition 2.1. Let $T : X \to E$ be an operator between Banach spaces.
(a) T is called almost narrow (or strong Daugavet operator) if for every two elements $x, y \in S_X$ and every $\varepsilon > 0$ there is some $z \in B_X$ such that $\|T(y - z)\| \leq \varepsilon$ and $\|x + z\| \geq 2 - \varepsilon$.
(b) T is called narrow if for every functional $x^* \in X^*$ the operator $T \oplus x^* : X \to E \oplus_1 \mathbb{R}$ defined by

$$(T \oplus x^*)(x) = (T(x), x^*(x))$$

is almost narrow.
definition 3.1. A subspace Y of a Banach space X is called rich (respect. almost rich) if the quotient map from X onto X/Y is narrow (respect. almost narrow).
We say that a subspace Y of a Banach space X with the DP is wealthy if Y and every subspace of X containing Y have the DP.

2 Main results

Clearly, every narrow operator is almost narrow. By [1] if X has the DP, then the narrow and weakly compact operators on X are equivalent. The following example shows that for

*Speaker