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Non-linear Semigroups in Hadamard Spaces

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Abstract

There are at least two methods to generate a non-linear semigroup of non-expansive operators in Hadamard spaces: gradient flows of convex maps and semigroups generated by m-co-accretive operators. Using an inner product-like notion of quasilinearization, we have established a *link* between these two approaches. We prove that in each geodesically unbounded Hadamard space X, each convex map $f : X \to (-\infty, +\infty]$ induces a co-accretive operator $T_f : X \to 2^X$ such that it generates a nonlinear semigroup which coincides the gradient flow of f.

Keywords: Hadamard space, non-linear semigroup, co-accretive operator, gradient flow, quasilinearization.
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1 Introduction

1.1 Hadamard Space

A $CAT(0)$ space is a metric space (X, d) such that for each two points $x_0, x_1 \in X$ and for each $0 < t < 1$ there exists some $x_t \in X$ such that

$$d^2(y, x_t) \leq (1-t)d^2(y, x_0) + td^2(y, x_1) - t(1-t)d^2(x_0, x_1) \quad (y \in X). \tag{1}$$

It can be seen that such x_t must be unique, so one can write $(1 - t)x_0 \oplus tx_1 = x_t$. A complete $CAT(0)$ space is called a *Hadamard* space. These spaces are well-studied by many authors; we refer the reader to the standard texts such as [5, 6]. There are many various examples of Hadamard spaces: Hilbert spaces, Hadamard manifolds (i.e., simply-connected complete Riemannian manifolds with nonpositive sectional curvature which can be of infinite dimension), any bounded domain in a complex Banach space with Carathéodory metric, e.g., open unit ball of a complex Hilbert space with Poincaré metric, $\mathbb{R}$-trees as well as examples that have been built out of given Hadamard spaces as: closed convex subsets, direct products, warped products, L^2-spaces, direct limits and Reshetnyak's gluing.

1.2 Co-accretive Operator

A Hadamard space (X, d) is called *geodesically unbounded* if for each $x, y \in X$ there exists a geodesic line $c : \mathbb{R} \to X$ passing through x, y, i.e., $d(c(t), c(s)) = |t - s|d(x, y)$ for $t, s \in \mathbb{R}$, $c(0) = x$ and $c(1) = y$. Every geodesically unbounded Hadamard space is