The BSE property of semigroup algebras

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Abstract

The concepts of BSE property and BSE algebras were introduced and studied by Takahasi and Hatori in 1990 and later by Kaniuth and Ülger. This abbreviation refers to a famous theorem proved by Bochner and Schoenberg for $L^1(\mathbb{R})$, where $\mathbb{R}$ is the additive group of real numbers, and by Eberlein for $L^1(G)$ of a locally compact abelian group G. In this paper we investigate the BSE property for certain semigroup algebras.

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1 Introduction

Let A be a commutative Banach algebra. Denote by $\Delta(A)$ and $\mathcal{M}(A)$ the Gelfand spectrum and the multiplier algebra of A, respectively. A bounded continuous function σ on $\Delta(A)$ is called a *BSE-function* if there exists a constant $C > 0$ such that for every finite number of $\varphi_1, ..., \varphi_n$ in $\Delta(A)$ and complex numbers $c_1, ..., c_n$, the inequality

$$\left|\sum_{j=1}^{n} c_j\sigma(\varphi_j)\right| \leq C. \left\|\sum_{j=1}^{n} c_j\varphi_j\right\|_{A^*}$$

holds. The BSE-norm of σ ($\|\sigma\|_{BSE}$) is defined to be the infimum of all such C. The set of all BSE-functions is denoted by $C_{BSE}(\Delta(A))$. Takahasi and Hatori [9] showed that under the norm $\|.\|_{BSE}$, $C_{BSE}(\Delta(A))$ is a commutative semisimple Banach algebra.

A bounded linear operator on A is called a *multiplier* if it satisfies $xT(y) = T(xy)$ for all $x, y \in A$. The set $\mathcal{M}(A)$ of all multipliers of A is a unital commutative Banach algebra, called the *multiplier algebra* of A.

For each $T \in \mathcal{M}(A)$ there exists a unique continuous function $\widehat{T}$ on $\Delta(A)$ such that $\widehat{T(a)}(\varphi) = \widehat{T}(\varphi)\hat{a}(\varphi)$ for all $a \in A$ and $\varphi \in \Delta(A)$. See [6] for a proof.

Define

$$\widehat{\mathcal{M}(A)} = \{\widehat{T} : T \in \mathcal{M}(A)\}.$$

A commutative Banach algebra A is called without order if $aA = \{0\}$ implies $a = 0$ $(a \in A)$.

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