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Class preserving automorphisms of finite p-groups

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Abstract

Let G be a finite non-abelian p-group and $\mathrm{Aut}_c(G)$ denote the group of all class preserving automorphisms of G. In this paper, using the notion of Frattinian groups, we give necessary condition for finite p-groups G for the groups $\mathrm{Aut}_c(G)$ and $\mathrm{Inn}(G)$ coincide when $(G, Z(G))$ is a Camina pair.

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1 Introduction

Let G be a finite p-group. For $x \in G$, x^G denotes the conjugacy class of x in G. By $\mathrm{Aut}(G)$ we denote the group of all automorphisms of G. An automorphism α of G is called class preserving if $\alpha(x) \in x^G$ for all $x \in G$. We let $\mathrm{Aut}_c(G)$ denote the set of all class preserving automorphisms of G. The group $\mathrm{Aut}_c(G)$ have been studied by several authors, see for example [3, 4, 10], [12, 13] . It is well known that if G is a finite p-group, then so is the group $\mathrm{Aut}_c(G)$, In this paper we study closely the groups $\mathrm{Aut}_c(G)$ for a finite non-abelian p-group G. We give necessary condition for finite p-groups G for the groups $\mathrm{Aut}_c(G)$ and $\mathrm{Inn}(G)$ coincide when $(G, Z(G))$ is a Camina pair. Throughout the paper all groups are assumed to be finite groups.

2 Main results

In this section we give some known results which will be used in the rest of the paper.

Let G be a finite p-group. Following Schmid, we call G Frattinian provided $Z(G) \neq Z(M)$ for all maximal subgroups M of G. In [11], P. Schmid proved the following structural theorem for the Frattinian groups.

Theorem 2.1 ([11])**.** *Suppose G is a non-abelian Frattinian p-group. Then one of the following holds:*

(i) G is the central product of non-abelian p-groups of order $p^2|Z(G)|$, amalgamating their centres.

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