Discrete mollification method and its application to solving backward nonlinear cauchy problem

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Abstract

In this article a nonlinear backward cauchy problem consisting of two unknown functions is considered. A space marching algorithm based on discrete mollification method is presented to solve this problem. Finally we illustrate some numerical examples to show efficiency of the proposed method.

Keywords: Nonlinear backward cauchy problem, Space marching algorithm, Discrete mollification

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1 Introduction

Consider a nonlinear backward inverse problem governed by

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}((a(x)+b(x)u^2)\frac{\partial u}{\partial x})+f(x,t);\quad 0<x<1,\quad 0<t<T, \tag{1}$$

$$u(x,T)=\varphi(x);\quad 0\le x\le 1, \tag{2}$$

$$u(0,t)=g_1(t);\qquad 0\le t\le T, \tag{3}$$

$$u_x(0,t)=g_2(t);\qquad 0\le t\le T, \tag{4}$$

where f(x,t), $a(x)>0, b(x), \varphi(x), g_1(t)$ and $g_2(t)$ are known.We are going to determine $u(x,t)$ and $u(x,0)$ satisfying (1)-(4). Now, we add random noise ,with maximum level of ε, in the initial data $\varphi(x), g_1(t)$ and $g_2(t)$. These noisy data are represented by $\varphi^{\varepsilon}(x), g_1^{\varepsilon}(t)$ and $g_2^{\varepsilon}(t)$,respectively. The particular difficulty of the backward problem is its ill-possedness, on the other hand since we have noise in the problem's data so should first regularize this problem by discrete mollification method [2]. The stabilized problem is described as

$$\frac{\partial v}{\partial t}=\frac{\partial}{\partial x}[(a(x)+b(x)v^2)\frac{\partial v}{\partial x}]+f(x,t);\quad 0<x<1,\quad 0<t<T, \tag{5}$$

$$v(x,T)=J_{\delta_1}\varphi^{\varepsilon}(x);\quad 0\le x\le 1, \tag{6}$$

$$v(0,t)=J_{\delta_2}g_1^{\varepsilon}(t);\quad 0\le t\le T, \tag{7}$$

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