



Representations of Polygroups Based on Krasner Hypervector Spaces

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Abstract

In this paper we introduce representations of polygroups by Krasner hypervector spaces. The goal of polygroup representation is to study polygroups via their actions on Krasner hypervector spaces. By acting on Krasner hypervector spaces even more detailed information about a polygroup can be obtained.

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1 Introduction

In [8] M. Motameni, R. Ameri and R. Sadeghi studied hypermatrix based on hyperspaces. The goal of representation of polygroups is to study polygroups via their actions on hyperspaces. By acting on hyperspaces even more detailed information about a polygroup can be obtained. In this note we introduced and study the representation of polygroups by Krasner hyperspaces and obtain some related basic results.

Recall that for a non-empty set H a *hyperoperation* or a *join operation* is a map $\cdot : H \times H \rightarrow P_*(H)$, where $P_*(H)$ is the set of all non-empty subsets of H .

Definition 1.1. [4] A polygroup is a special case of a hypergroup. A polygroup is a system $\mathcal{P} = \langle P, \cdot, e, {}^{-1} \rangle$, where $e \in P$, ${}^{-1}$ is a unary operation on P , \cdot maps $P \times P$ into nonempty subsets of P , and the following axioms hold for all $x, y, z \in P$:

- (P₁) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$,
- (P₂) $x \cdot e = e \cdot x = x$,
- (P₃) $x \in y \cdot z$ implies $y \in x \cdot z^{-1}$ and $z \in y^{-1} \cdot x$.

Definition 1.2. [3] A Krasner hyperring is a hyperstructure (R, \oplus, \star) where

- (i) (A, \oplus) is a canonical hypergroup;
- (ii) (A, \star) is a semigroup endowed with a two-sided absorbing element 0;
- (iii) the product distributes from both sides over the sum.

Definition 1.3. [3] Let (K, \oplus, \star) be a hyperfield and (V, \oplus) be a canonical hypergroup. We define a Krasner hyperspace over K to be the quadrupled (V, \oplus, \cdot, K) , where \cdot is a single-valued operation

$$\cdot : K \times V \rightarrow V,$$

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