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Open questions concerning Hindman's theorem

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Abstract

Hindman's theorem states that for every coloring of $\mathbb{N}$ with finitely many colors, there is an infinite set A such that the set of numbers which can be written as a sum of distinct elements of A is monochromatic. In this paper, we survey some interesting questions concerning this theorem.

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1 Introduction

Let $\mathbb{N}$ be the set of nonnegative integers. Given $X \subseteq \mathbb{N}$ let $FS(X)$ be the set of all sums of finite nonempty subsetes of X. Hindma's theorem is the following statement.

Theorem 1.1. *(Hindman) If $\mathbb{N} = C_0 \cup \ldots \cup C_l$, then there exists an infinite set $X \subseteq \mathbb{N}$ such that $FS(X) \subseteq C_i$ for some $i \leq l$.*

There are four proofs of Hindman's theorem:
(1) The original combinatorial proof due to Hindman [6];
(2) The simplified combinatorial proof due to Baumgartner [1];
(3) The dynamical proof due to Furstenberg and weiss [2];
(4) The ultrafilter proof due to Glazer [4].

The notion of an ultrafilter is a powerful tool in set theory, combinatorics and topology. We here give a short proof of Hindman's theorem using ultrafilters. For more details see [3]. An ultrafilter on a set X is a set of subsets $\mathcal{F} \subseteq \mathcal{P}(X)$ satisfying

1. $X \in \mathcal{F}$ and $\emptyset \notin \mathcal{F}$.
2. If $A \in \mathcal{F}$ and $B \in \mathcal{F}$ then $A \cap B \in \mathcal{F}$.
3. For all $A \subseteq X$, either $A \in \mathcal{F}$ or $A^c \in \mathcal{F}$.

Theorem 1.2. *Let $\mathcal{F}$ be an ultrafilter on a set X.*
1. *If B is such that $A \cap B \neq \emptyset$ for all $A \in \mathcal{F}$ then $B \in \mathcal{F}$.*
2. *If A and B are such that $A \cup B \in \mathcal{F}$ then at least one of $A, B \in \mathcal{F}$.*

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