A smooth approximation for numerical solution of nonlinear Schrödinger equations

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Abstract

We present a numerical method based on exponential splines for solving the nonlinear Schrödinger equations with variable coefficients. The error analysis, stability and convergence properties of the method are investigated. The efficiency of the method is demonstrated by test problems. The numerical simulations validate and demonstrate the advantages of the method.

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1 Introduction

We consider the following nonlinear Schrödinger

$$\imath\frac{\partial u}{\partial t}+\alpha(t)\frac{\partial^2 u}{\partial x^2}+F(x,t)u+\beta(t)|u|^2u=0, \quad a<x<b, \quad 0<t\le T, \tag{1}$$

with the boundary conditions

$$u(a,t)=f_0(t), \quad u(b,t)=f_1(t), \quad 0<t\le T, \tag{2}$$

and the initial condition

$$u(x,0)=\phi(x), \quad x\in[a,b], \tag{3}$$

where $\alpha(t)$, $F(x,t)$ and $\beta(t)$ are bounded real functions and also $u(x,t)$ is the complex-valued wave function and $\alpha(t)$ is related to the second order dispersion coefficient.This equation is one of the most universal models that describes many physical nonlinear systems. This problem has been studied by several authors such as [1, 2].

The purpose of this paper is to give a numerical method, based on a uniform mesh using exponential splines for the nonlinear Schrödinger equation, which leads to the recurrence relation. We give the truncation error of the method, convergence and stability analysis. The analysis will be illustrated by investigating some examples. The numerical simulations validate and demonstrate the advantages of the method.

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