Constructing dual and approximate dual fusion frames

Fahimeh Arabyani*
Hakim Sabzevari University

Ali Akbar Arefijamaal
Hakim Sabzevari University

Abstract

The main goal of this paper is the construction of dual and approximate dual fusion frames. We introduce the notion of approximate duality for fusion frames, and present some approaches to obtain dual fusion frames. In particular, we characterize all duals of a Riesz decomposition fusion frame.

Keywords: Fusion frames; dual fusion frames; approximate duals; Riesz decomposition
Mathematics Subject Classification [2010]: 42C15

1 Introduction

In this section we review some definitions and primary results of fusion frames and show that, unlike discrete frames, every fusion frame has at least one alternate dual. Throughout this paper, π_V denotes the orthogonal projection from $\mathcal{H}$ onto a closed subspace V.

Definition 1.1. Let $\{W_i\}_{i\in I}$ be a family of closed subspaces of $\mathcal{H}$ and $\{\omega_i\}_{i\in I}$ be a family of weights, i.e. $\omega_i > 0$, $i \in I$. Then $\{(W_i, \omega_i)\}_{i\in I}$ is called a *fusion frame* for $\mathcal{H}$ if there exist the constants $0 < A \leq B < \infty$ such that

$$A\|f\|^2 \leq \sum_{i\in I} \omega_i^2 \|\pi_{W_i} f\|^2 \leq B\|f\|^2, \qquad (f \in \mathcal{H}). \tag{1}$$

The constants A and B are called the *fusion frame bounds*. If we only have the upper bound in (1) we call $\{(W_i, \omega_i)\}_{i\in I}$, a *Bessel fusion sequence*. A fusion frame is called A-*tight*, if $A = B$, and *Parseval* if $A = B = 1$. If $\omega_i = \omega$ for all $i \in I$, the collection $\{(W_i, \omega_i)\}_{i\in I}$ is called ω-*uniform* and we abbreviate 1- uniform fusion frames as $\{W_i\}_{i\in I}$. A fusion frame $\{W_i\}_{i\in I}$ is called an orthonormal basis for $\mathcal{H}$ when $\oplus_{i\in I} W_i = \mathcal{H}$ and it is a *Riesz decomposition* of $\mathcal{H}$ if for every $f \in \mathcal{H}$, there is a unique choice of $f_i \in W_i$ such that $f = \sum_{i\in I} f_i$. It is clear that every orthonormal fusion basis is a Riesz decomposition for $\mathcal{H}$, and also every Riesz decomposition is a 1- uniform fusion frame for $\mathcal{H}$.

Let $\{(W_i, \omega_i)\}_{i\in I}$ be a fusion frame, the *fusion frame operator* $S_W : \mathcal{H} \to \mathcal{H}$ is defined by $S_W f = \sum_{i\in I} \omega_i^2 \pi_{W_i} f$ is a bounded, invertible as well as positive. Hence, we have the following reconstruction formula [4]

$$f = \sum_{i\in I} \omega_i^2 S_W^{-1} \pi_{W_i} f, \qquad (f \in \mathcal{H}).$$

*Speaker