On Chatterjea Contractions in Metric Space with a Graph

Kamal Fallahi*
Department of Mathematics, Payame Noor University, I.R. of IRAN

Abstract

In this talk, we introduce Chatterjea contractions using directed graphs in metric spaces with a graph and investigate the existence of fixed points for Chatterjea contractions under two different conditions and discuss the main theorem. We also discuss the uniqueness of the fixed point.

Keywords: G-Chatterjea mapping, Fixed point, Orbitally G-continuous mapping.
Mathematics Subject Classification [2010]: 47H10, 05C20

1 Introduction

Let (X, d) be a metric space. In [3], Chatterjea introduced the notion of Chatterjea contraction on a metric space X as follows:

$$d(Tx, Ty) \leq \alpha\big[d(x, Ty) + d(y, Tx)\big] \tag{1}$$

for all $x, y \in X$, where $\alpha \in [0, \frac{1}{2})$. He also investigated the existence and uniqueness of fixed points for self-map T and proved that such mappings have a unique fixed point in complete metric spaces.

Recently in 2008, Jachymski [4] proved some fixed point results in metric spaces endowed with a graph and generalized simultaneously the Banach contraction principle from metric and partially ordered metric spaces. Recently in 2013, Bojor [1] followed Jachymski's idea for Kannan contractions using a new assumption called the weak T-connectedness of the graph.

The aim of this paper is to study Chatterjea contractions in metric spaces endowed with a graph by standard iterative techniques and avoid imposing the assumption of weak T-connectedness on the graph. Our main result generalizes Chatterjea's fixed point theorem in metric spaces and also in metric spaces equipped with a partial order.

We next review some basic notions of graph theory in relation to uniform spaces that we need in the sequel. For more details on the theory of graphs, see, [2, 4].

An edge of an arbitrary graph with identical ends is called a loop and an edge with distinct ends is called a link. Two or more links with the same pairs of ends are said to be parallel edges.

Let (X, d) be a metric space and G be a directed graph with vertex set $V(G) = X$ such that the set $E(G)$ consisting of the edges of G contains all loops, that is, $(x, x) \in E(G)$

*Speaker