# Application of coupled fixed point theorems in partially ordered sets to a boundary value problem of fractional order

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**Abstract**

In this paper, we discuss the existence and uniqueness of solutions for nonlinear boundary value problem of differential equations of fractional order. Our analysis relies on the coupled fixed point theorems in partially ordered metric spaces.

**Keywords:** Partially coupled fixed point, Caputo fractional differential equation, Existence, Uniqueness.
**Mathematics Subject Classification [2010]:** 26A33, 34A08, 34A12.

## 1 Introduction

Fractional differential equations have attracted huge attention in the past few years because of their unique physical properties and their potential in the modeling of many physical phenomena and also in various field of science and engineering [3, 2, 5]. During last years, the study of such kind of problems have received much attention from both theoretical and applied point of view. We will mention the following recent works on this topic [3, 2, 5, 6, 7].

In this paper, we study the following boundary value problems (BVP) for fractional differential equations involving the Caputo derivative

$$\begin{cases} (D_*^\alpha y)(x) = f(x, y(x)), & (0 < \alpha < 1, x \in [0, T]), \\ ay(0) + by(T) = c, \end{cases} \tag{1}$$

where $D_*^\alpha$ is the Caputo fractional derivative of order $\alpha$, $f : [0, T] \times \mathbb{R} \to \mathbb{R}$ is a given function satisfying some assumptions that will be specified later and $a, b, c \in \mathbb{R}$ are real constants with $a + b \neq 0$.

The existence of solutions for this kind of BVP has been studied by several authors, see [2, 4]. We will present the new existence and uniqueness results for the fractional BVP (1) using partially couple fixed point theorems. The advantage and importance of this method arises from the fact that it is a constructive method that yields monotone sequences that converge to the unique solution of BVP (1).

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