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Properties Hypergeometric Functions by Ruscheweyh Derivative*

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Abstract

We study properties of starlike and convexity for the hypergeometric function F(a, b; c; z) defined by Ruscheweyh derivative through putting conditions on a, b, c, to ensure that zF(a, b; c; z) will be in various subclasses of starlike and convex functions.

Keywords: Starlike, Convex , Ruscheweyh Derivative, Hypergeometric functions.
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1 Introduction

let S denote the class of all functions f of the form

$$f(z) = z + \sum_{n=0}^{\infty} a_n z^n \tag{1}$$

that are analytic and univalent in the open unit disk $\Delta = \{z \in C :| z |< 1\}$.

Definition 1.1. A function $f \in S$ is said to be starlike of order $\beta(0 \leq \beta < 1)$ if and only if $Re\Big(\frac{zf'(z)}{f(z)}\Big) > \beta$.
Denote the class of all starlike functions of order β in Δ by $S^{\star}(\beta)$.

Definition 1.2. A function $f \in S$ is said to be convex of order $\beta(0 \leq \beta < 1)$ if and only if $Re\Big(\frac{1+zf''(z)}{f'(z)}\Big) > \beta$.
Denote the class of all convex functions of order β in Δ by $C(\beta)$.

Let (a,n) denote symbol for the generalized factorial ,

$(a, 0) = 1$ for $a \neq 0$, $(a, n) = a(a + 1)(a + 2)...(a + n - 1)$ for $n \in N$.

and the Gaussian hypergeometric function given by the analyti function ,

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