On linear operators from a Banach space to analytic Lipschitz spaces

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Abstract

In this note, we characterize boundedness and (weak) compactness of linear operators from a Banach space into analytic Lipschitz spaces $\mathrm{lip}_A(X,\alpha)$. We also obtain a lower bound for the essential norm of such operators.

Keywords: Analytic Lipschitz algebra;compact linear operator; weakly compact linear operator; essential norm.

Mathematics Subject Classification [2010]: 47B38, 46E15

1 Introduction

Let E be a Banach space, (X,d) be a compact metric space, and $\alpha \in (0,1]$. The space $\mathrm{Lip}_\alpha(X,E)$ consist of E-valued functions f on X that

$$p_\alpha(f) = \sup\left\{\frac{\|f(x)-f(y)\|_E}{d^\alpha(x,y)} : x,y \in X, x \neq y\right\} < \infty,$$

and $\mathrm{lip}_\alpha(X,E)$ is the subspace of those functions f for which

$$\lim_{d(x,y)\to 0} \frac{\|f(x)-f(y)\|_E}{d^\alpha(x,y)} = 0.$$

The spaces $\mathrm{Lip}_\alpha(X,E)$ and $\mathrm{lip}_\alpha(X,E)$ are Banach spaces with the norm $\|f\|_\alpha = \|f\|_X + p_\alpha(f)$, where $\|f\|_X = \sup_{x\in X}\|f(x)\|_E$. In the case that E is the scalar field of the complex numbers $\mathbb{C}$, we have classic Lipschitz algebras $\mathrm{Lip}(X,\alpha) = \mathrm{Lip}_\alpha(X,\mathbb{C})$ and $\mathrm{lip}(X,\alpha) =$

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