



## J-Armendariz Rings Relative to a Monoid

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### Abstract

For a monoid  $M$ , we introduce J-M-Armendariz rings, which is a common generalization of J-Armendariz and weak M-Armendariz rings, and investigate their properties. We show that every NI-ring is J-M-Armendariz, for any unique product monoid  $M$ . Also, we provide various examples and classify how the J-M-Armendariz rings behave under various ring extensions. It is shown that if  $R$  is semicommutative ring and M-Armendariz then  $R$  is J-( $M \times N$ )-Armendariz, where  $N$  is a unique product monoid.

**Keywords:** J-M-Armendariz ring; Semicommutative ring; Jacobson radical; u.p.-monoid.

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## 1 Introduction

Throughout this paper every ring is an associative ring with identity unless otherwise stated. For a ring  $R$ , we denote by  $nil(R)$  the set of all nilpotent elements of  $R$  and by  $J(R)$  the Jacobson radical of  $R$ . The  $n$ -by- $n$  full (resp. upper triangular) matrix ring over  $R$  is denoted by  $Mat_n(R)$  (resp.  $T_n(R)$ ), and  $E_{ij}$ 's denote the matrix units.  $\mathbb{Z}$  and  $\mathbb{C}$  denote the ring of integers and the field of complex numbers. The polynomial ring with an indeterminate  $x$  over  $R$  is denoted by  $R[x]$ . A ring  $R$  is said to be Armendariz if the product of two polynomials in  $R[x]$  is zero if and only if the product of their coefficients is zero. This definition was coined by Rege and Chhawchharia in [1] in recognition of Armendariz's proof in [2] that reduced rings (i.e., rings without nonzero nilpotent elements) satisfy this condition. Recently, several types of generalizations of Armendariz rings have been introduced (see, e.g., [3, 4, 5]). Liu and Zhao [4], studied the structure of the set of nilpotent elements in Armendariz rings and introduced weak Armendariz rings as a generalization. A ring  $R$  is said to be weak Armendariz ring if the product of two polynomials in  $R[x]$  is zero, then the product of their coefficients is nilpotent. C. Zhang and J. Chen [5], studied a generalization of weak Armendariz rings, which is called weak M-Armendariz rings. A ring is called weak M-Armendariz (weak Armendariz relative to  $M$ ) if whenever  $\alpha = a_1g_1 + \cdots + a_n g_n, \beta = b_1h_1 + \cdots + b_m h_m \in R[M]$ , with  $g_i, h_j \in M$

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