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Numerical solution of fractional Fokker-Planck equation by using of radial basis functions

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Abstract

In this paper, we propose a numerical method which is coupled of the radial basis functions (RBFs) and finite difference scheme for solving time fractional Fokker-Planck equation defined by Caputo sense for $(0 < \alpha < 1)$. It uses the collocation method and approximates the solution using thin plate splines (TPS) RBFs.

Keywords: Fractional differential equation, Fokker-Planck equation, Radial Basis Functions(RBFs), Collocation method
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1 Introduction

The Fokker-Planck equation (FPE) was first introduced by Fokker and Planck to describe the Brownian motion of particles. Phenomena such as anomalous diffusion, continuous random walk, wave propagation and etc. are modeled by space and time fractional FPE (see [4] and references therein.).

Consider the following time fractional FPE of order α $(0 < \alpha < 1)$ with the initial and boundary conditions:

$$D_t^{\alpha} u - u_{xx} + p(x)u_x + p'(x)u = f, \qquad 0 < x < L, 0 < t \le T, \tag{1}$$

$$u(x,0) = g(x), \qquad 0 \le x \le L, \tag{2}$$

$$u(0,t) = h_1(t), \qquad u(L,t) = h_2(t), \qquad 0 < t \le T. \tag{3}$$

where D_t^{α} is the Caputo fractional derivative operator of order $\alpha \ge 0$, which is defined as

$$D_t^{\alpha} u(x,t) = \frac{1}{\Gamma(k-\alpha)} \int_0^t (t-s)^{k-\alpha-1} \frac{\partial^k u(x,s)}{\partial s^k} ds, \qquad k-1 < \alpha < k. \tag{4}$$

The fractional FPE has been solved in several ways including (high–order) finite difference methods [1, 5] and finite element method [2].

Considering a finite set of interpolation points $\chi = \{x_1, x_2, \ldots, x_N\} \subset \mathbb{R}^d$ and a function $u : \chi \to \mathbb{R}$, the interpolant of u using radial basis functions (RBFs) is constructed as

$$u(x) \simeq \sum_{i=1}^{N} \lambda_i \phi(\|x - x_i\|) + \sum_{j=N+1}^{N+\ell} \lambda_j q_j(x), \qquad x \in \mathbb{R}^d, \ \ell = \binom{m+d-1}{d} \tag{5}$$

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