# Mittag-Leffler identity for half-Hermite transform

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**Abstract**

In this paper in view of the Fourier series of a periodic function on interval $(0, \infty)$, we obtain a Mittag-Leffler type identity for the half-Hermite transform of order $n$.

**Keywords:** Mittag-Leffler identity, Fourier series, Half-Hermite transform
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## 1 Introduction and Preliminaries

We consider the periodic function $f(x)$ and approximate it by a Fourier series with period $2T$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(\frac{2n\pi x}{T}) + b_n \sin(\frac{2n\pi x}{T})], \tag{1}$$

where $a_n$ and $b_n$ are the Fourier coefficients as follows

$$a_n = \frac{1}{T}\int_0^{2T} f(x)\cos(\frac{2n\pi x}{T})dx, \quad n = 0, 1, 2, \cdots, \tag{2}$$

$$b_n = \frac{1}{T}\int_0^{2T} f(x)\sin(\frac{2n\pi x}{T})dx, \quad n = 1, 2, \cdots. \tag{3}$$

Related to the theory of integral transforms, by applying the suitable integral transform on relation (1), the Mittag-Leffler identity can be written. For example, using the Laplace transform this identity is obtained as [4]

$$\frac{a_0}{2s} + T\sum_{n=1}^{\infty}\frac{sTa_n + 2\pi n b_n}{s^2T^2 + 4\pi^2 n^2} = \frac{1}{1 - e^{-sT}}\int_0^T e^{-su} f(u), \tag{4}$$

and using the Meijer transform, we get [3]

$$\frac{a_0\pi}{4s} + \sum_{n=1}^{\infty}[\frac{\pi T a_n}{2\sqrt{n^2\pi^2 + s^2T^2}} + \frac{Tb_n}{\sqrt{n^2\pi^2 + s^2T^2}}\ln\left(\frac{n\pi}{Ts} + \sqrt{\frac{n^2\pi^2}{T^2s^2} + 1}\right)] =$$
$$\int_0^T f(x)[K_0(sx) + \int_0^\infty \frac{1}{\sqrt{t^2+s^2}}\frac{e^{-xt}}{e^{Tt} - 1}dt]dx, \tag{5}$$

where $K_0$ is the modified Bessel function of second kind.

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