First hochchild cohomology of square algebra

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Abstract

In this paper, we define the square algebra and describe the first hochschild cohomology of this algebra.

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1 Introduction

If $\mathbb{A}$ and $\mathbb{B}$ are algebras, M is an A , B-module. and N is a B , A-module, then we will call $S = \begin{bmatrix} A & M \\ N & B \end{bmatrix}$ a square algebra. We study the structure of Hochschild cohomology groups of square algebra. This groups is important in many areas of mathematics, such as ring theory, commutative algebra, geometry, group theory and etc.
Although Hochschild cohomology for algebras has been studied extensively for many years, there are still few techniques available for explicitly calculating the various cohomology groups. The study of first cohomology group $H^1(A, X)$, where A is algebra and X is A-bimodule, is essentially the study of inner dervations.

2 Main results

Definition 2.1. Let A and B be algebra. Let M be an A, B-module and N be a B, A-module such that $M \otimes_B N = 0 = N \otimes_A M$. We put
$$S = \left\{ \begin{bmatrix} a & m \\ n & b \end{bmatrix} : a \in A, m \in M, n \in N, b \in B \right\}.$$

If S is given the usual operations associated with 2×2 matrices, then S becomes an algebra. We shall call such an algebra a square algebra.
If A is algebra, a continuous derivation on A is a bounded linear operator $S : A \longrightarrow A$ such that $\delta(ab) = a\delta(b) + \delta(a)b$. Given $x \in A$, we define the map $\delta_x : A \longrightarrow A$ by
$\delta_x(a) = xa - ax$.
The map δ_x is easily seen to be a continuous derivations are said to be inner. Let Der (A) denote all continuous derivation of A and Let Inn (A) denote all inner derivations. We define $H^1(A, A)$, the first cohomology group of A by
$H^1(A, A) = Der(\text{A})/Inn(\text{A})$.

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